Ver May 2018

Galileo, the First Amateur Telescope Maker

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ABSTRACT

Galileo Galilei did not invent the telescope which carries his name, but he did have a great impact on its development into an astronomical instrument. Through his own reticence to speak about the fabrication, we have very little direct information about his methods and optical capabilities. This paper will discuss the so-called Galilean telescope and point out differences between the astronomical version and the original spyglass version. A simulation of one of the instruments in the *Museo Galileo* will be constructed and it will be used in a similar manner to Galileo, encountering the same ergonomic problems and observing limits. Some speculation about Galileo's testing methods will be made as well as conjectures about the tools of his workshop. Based on modern observations of the various targets through these telescopes and test apertures over larger instruments, the likeliest candidates in the *Museo Galileo* for the observations of *Sidereus Nuncius* will be identified. ZEMAX models will elucidate various points, and a more mathematical appendix is included. Although one can never be entirely sure about Galileo's methods, one can infer the connections between events that had to exist for him to be a successful telescope maker and observer.

1.0 INTRODUCTION

Somewhere around the year 1608, a new optical instrument burst forth upon the world. It seems to have taken the final steps in development from a toy to a useful instrument in the hands of spectacle-makers in the city of Middelburg, Zeeland. A few stories about a blurry enlargement being produced with substandard lenses existed earlier, and obscure descriptions might be later interpreted to have been reports of similar optical devices, but any real instruments that were attempted prior to this year must have been so pitiful that they didn't create the excitement of 1608.

What happened in 1608? Rolf Willach has proposed a hypothesis that merely stopping down the poorly-shaped spectacle lenses was responsible for converting a blurry optical curiosity into a commercial instrument. In his introduction to Willach's book, Albert Van Helden points out that in 1608 there weren't any telescopes, and in 1609 they were everywhere.[1] Clearly something happened in that year that offered an abrupt and easily-duplicated increase in quality, and Willach's guess of an aperture stop is plausible.[2]

Then, in 1610, a small book was published by a lesser-known mathematician at the University of Padua. The pamphlet-length publication that revolutionized astronomy was *Sidereus Nuncius* (in Latin, 1610). The translation of the title is *Starry Message* or *Messenger*. He vaulted into fame and controversy, and we still know the author best by his first name of Galileo. The telescope he described was radically different from the 3- or 4-power Dutch spyglass; it possessed larger aperture (for the time) and much higher magnification. Galileo used it to transform observational astronomy. Much has been made of his conflicts with the Church, but that will not be discussed here. The problem to be considered is his fabrication of the telescopes themselves and the methods he used to make them.

Most of the effort in Galileo studies has been based in traditional historical and comparative literature methods, sorting through the scraps of undiscarded paper left at the end of a long life of publishing and correspondence. What has resulted is an account of politics and maneuvering, with much discussion of who knew what (and when) but little technical detail. Galileo himself was partly responsible. For whatever reason, he didn't reveal particulars in the actual lens grinding and testing, maybe viewing them as unimportant or valuable shop secrets. Also, he may have been still in the scholastic-dialectic mode of thinking – popular at the time – where all truths had to emerge fully formed from the brows of advanced thinkers after a carefully reasoned and defined argument. For unknown reasons, Galileo published in his *Nuncius* a thin, watered-down description of his telescope making:

> About ten months ago a report reached my ears that ***a Dutchman had constructed a spyglass***, by the aid of which visible objects, although at a great distance from the eye of the observer, were seen distinctly as if near; and some demonstrations of its wonderful performances were reported, which some gave credence to, but others contradicted... [snip] ... This finally determined me to give myself up first to inquire into the principle of the spyglass, and then to consider the means by which I might arrive at the invention of a similar instrument. ***After a little while I succeeded, through deep study of the theory of refraction***. I prepared a tube, at first of lead, in the ends of which I fitted two glass lenses, both plane on one side, but on the other side one spherically convex, and the other concave. Then bringing my eye to the concave lens I saw objects satisfactorily large and near, ***for they appeared one-third of the distance*** and nine times larger than when they are seen with the natural eye alone. Shortly afterwards I constructed another more precise spyglass, which magnified objects more ***than 60 times***. Finally, by sparing neither labor

[1] Van Helden, foreword, in R. Willach, *The Long Road to the Invention of the Telescope*, p. vii, Am. Phil. Soc. (2008)

[2] For 3-power spyglasses, the incoming beam is already limited (in the daytime) to about 9 mm, but its position wanders freely on the front face of the objective lens depending on the centering and diameter of the eye's opening. Willach's hypothesis moves the aperture stop from an uncertain floating position at the pupil of the eye to a fixed hole at the center of the objective lens, where the optical quality is likely to be better and would certainly be more stable.

> nor expense, I succeeded in ***constructing for myself*** an instrument so superior that objects seen through it ***appear magnified nearly 1,000 times, and more than 30 times nearer than if viewed by the natural powers of sight alone***…[snip]…
>
> Here it is fitting that all who intend to turn their attention to observations of this kind should receive certain cautions. In the first place, it is absolutely necessary for them to prepare a most perfect spyglass, one that will show very bright objects distinct and free from any mistiness and will magnify them at least 400 times and ***show them as if only one-twentieth of their distance***. Unless the instrument be of such power, it will be in vain to attempt to view all the things that have been seen by me in the heavens, and that will be enumerated below.[3] [bold italics mine]

From this description, we can infer that he immediately constructed a 3-power instrument (or 3x) with a lead tube, based on descriptions of a Dutch original (he does not claim credit for the spyglass implementation). He points out the angular area of the image of the image has been stretched to 3x3 = 9 times the unaided area. Then he says that he constructed another spyglass which expanded the area about 60 times. Although he does not give the linear magnification, this telescope is just under 8 power. He gives no times or dates, but the implied haste indicates that he probably obtained the lenses (so far) from nearby spectacle makers. Then he says that he himself made an instrument that magnified the area over 1000 times. The square root of 1000 is 31.7 power, but it is safe to assume these measurements are somewhat inaccurate. He states that the readers must have good telescopes of at least 20x before they can duplicate the measurements contained in the *Nuncius*. Presumably he had also made a 20x, although in *Nuncius* this appears only as a recommendation.

He also made similar remarks in a later publication called *The Assayer*, in which he argued (not convincingly) that his development was more difficult than the invention of the original spyglass! Again he gave the steps that describe the way in the first few hours he would have inferred the way a spyglass worked,[4] but not how he later improved it at great "labor and expense" as he says in *Nuncius*.

Another place can you pin him down is a response to a question about the aperture stop on large lenses from Jesuit scholar Christoph Clavius:

> The first is to make it possible to work it more accurately because a large surface is more easily kept in the proper shape than a smaller one. The other reason is that if one wants to see a larger space in one glance, the glass can be uncovered, but it is then necessary to put a less acute glass near the eye and shorten the tube, otherwise the objects will appear very fuzzy.[5]

What he is saying here is that the large optical piece is better supported on the tool during grinding and polishing than a small piece is. If a larger view is desired, he says the maker must open the aperture and put in a longer focus eyepiece, cutting the tube shorter to reach focus (by the way, the "uncover glass" procedure only works well to expand the field of low-power spyglasses – see Appendix). Other than a letter where he discussed why he had an oval lens mask, he wrote little else that survives.

What I'm going to do in these pages is attempt to infer what may have happened during the early months of Galileo's development of the astronomical telescope. I will not be sifting through old records, weighing the opinions of one Renaissance worthy against another. I will be doing a process of reverse engineering, trying it out and relying on the fact that the physical world has not changed since Galileo's time. I will be trying to guess what technologies Galileo used to design and fabricate his telescopes. To do this, I will assemble and test a 20x instrument. More importantly, I will use it, encountering the same sort of

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3 Galileo Galilei, "The Sidereal Messenger (1610)," *The Essential Galileo*, ed. and trans., Maurice A. Finocchiaro, Hackett Publishing, Indianapolis/Cambridge, p. 45+, 2008.

4 excerpt from Galileo's "The Assayer" in *Discoveries and Opinions of Galileo*, ed., trans., and with notes by Stillman Drake, pp. 244-245, Doubleday-Anchor, 1957 (orig 1623).

5 Reprinted in Albert Van Helden, "Galileo and the Telescope," in *Origins of the Telescope,* Edited by Albert Van Helden *et al*., p. 192, Royal Netherlands Academy of Arts and Sciences, KNAW Press, 2010.

problems that Galileo did. In some explanations, I sometimes use the terminology of modern optics, but I will not impose them anachronistically on the actions of the 17th century opticians.

2.0 JUST WHAT IS A GALILEAN TELESCOPE?

A "Galilean telescope" is more of a description of the eyepiece of these early telescopes than the whole telescope, like a "Dobsonian telescope" more accurately refers to the mounting. Coupled with a long-focus positive objective lens is a stronger negative-focus ocular. It automatically has an upright image. A positive ocular is called the "Keplerian eyepiece," after Johannes Kepler, who in 1611 wrote the first modern work of geometric optics, *Dioptrice,* from which the positive eyepiece may be inferred. All modern astronomical telescopes use the upside-down Keplerian oculars, and the Galilean is considered a dead end in the evolution of the astronomical eyepiece.

With or without achromatic lenses, the negative-eyepiece form works best for instruments of power of 2 to 4. Galileans are made even up to the present in the form of straight-through field glasses and low-powered "opera" glasses. A miniaturized form of this very first telescope is still amazingly popular today in the form of tiny surgical or dental-hygienist magnifying loupes. These mini-telescopes are mounted on eye-shields or rings fitted around the brow and are closely focused about 0.33 to 0.5 meters in front of the user's face. No statistics exist on the use of such devices, but I imagine that the sum total of eyeball time spent with these small medical binoculars rivals or exceeds the visual use of all Keplerian binoculars or telescopes combined. Figure 1 shows my small Galilean binocular field glass, purchased in the 1990's, and an inexpensive set of medical eye loupes.

Figure 1. (left) A Russian miniature Galilean field glass: 2.5x17.5mm. (right) Medical Galilean eye loupe (~3x) focused at 320 mm

For much of the first 40 years, between 1608 and 1645, the hand-held telescope was exclusively of the Galilean type.[6] Figure 2 shows one reason why; the Galilean spyglass is much shorter. However, the Galilean has a smaller apparent field of view at any one time. At low power, this field is still large enough to be unobjectionable (see "Dutch" telescope fabrication at the end of the Appendix)

Wide apparent field was an attractive advantage, and Keplerian optics soon took over. The classic type of long, spindly, multiple-draw spyglass became the norm. The length of the Keplerian spyglass is derived from two factors. First, the positive (Keplerian) eyepiece is placed behind focus instead of in front of it. Second, there has to be a long image-erecting assembly placed between first focus and the Keplerian eyepiece. (The Galilean image is already upright.) For reasons that I won't go into here, good images result from the indicated dual lens image-flipping accessory instead of just one lens. Once prism binoculars were introduced to invert the image and fold the optical path in the late 19th century, the

[6] J. E. Greivenkamp and D.L. Steed, "The History of Telescopes and Binoculars: An Engineering Perspective, *Proc. SPIE Vol. 8129*, 812902, pp.1-18, 2011.

multiple-draw spyglass quickly lost popularity. However, it turns out that a long column of lenses may be mounted in such a manner that it can survive jerky accelerations and return repeatedly to the same position, so the long Keplerian spyglass is the form still preferred in riflescopes.

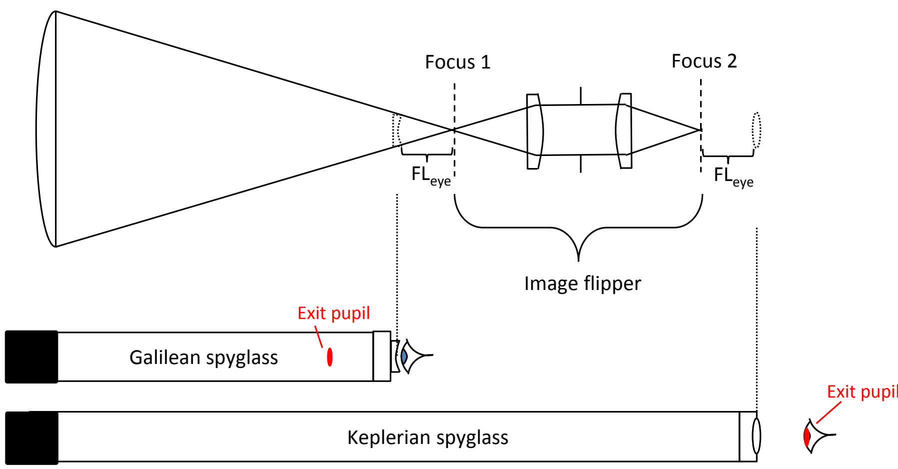


Figure 2: Difference in length of the Galilean-eyepiece telescope vs. Keplerian upright-image image telescope.

With both of these advantages of compactness and erect image, why is the Galilean relegated to specialty forms like the loupe? I have indicated the answer above in Figure 2 with the red graphics. The exit pupil of the Galilean telescope is in a virtual-image position deep within the interior. Even with the eyeball pushed uncomfortably up to the glass, users can't get close to the exit pupil. In the Keplerian configuration, this pupil is a real image of the objective floating behind the eyepiece and the eye's pupil opening may be placed right on top of it. The whole ocular lens, as viewed from the pupil, is filled with light. In the Galilean form, the eye may be moved around and a different tiny field will show, but one can never reach the exit pupil and light up the whole field of view. A modern name for this is *negative eye relief*. See Figure 3 for a ZEMAX diagram of the behavior of light rays around the eyepiece.[7]

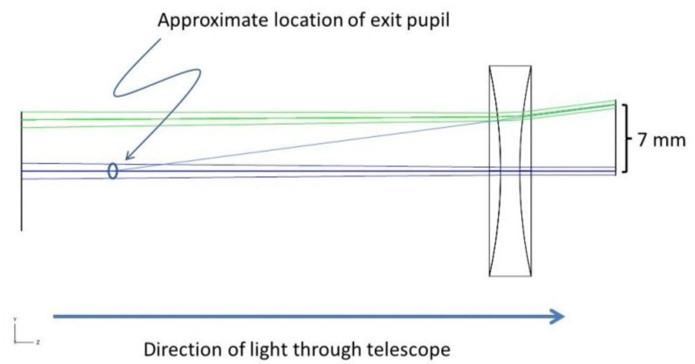
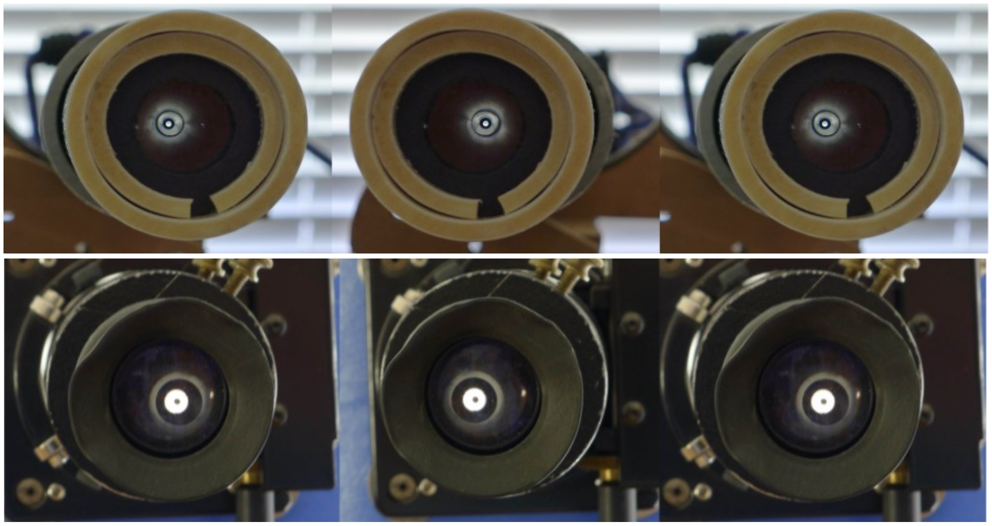


Figure 3. ZEMAX model of eyepiece end of Galileo's 20x telescope. The 7-mm eye opening is set off center, just as it would likely be in actual use. The exit pupil of the instrument is deep inside the instrument.

One can simulate the disadvantage of the Galilean configuration by deliberately drawing back several inches from the eyepiece in a modern positive-eyepiece telescope. The exit pupil shrinks until the telescope is almost unusable. That is the normal operating condition of a Galilean telescope! Field of view depends not only on the focal length of the objective but the diameter of the eye's pupil (please see the Appendix). It is only slightly annoying in a very low-power spyglass, but at the high power of Galileo's

[7] *ZEMAX-SE* , Optical design application, ZEMAX Corporation , April 14 2010

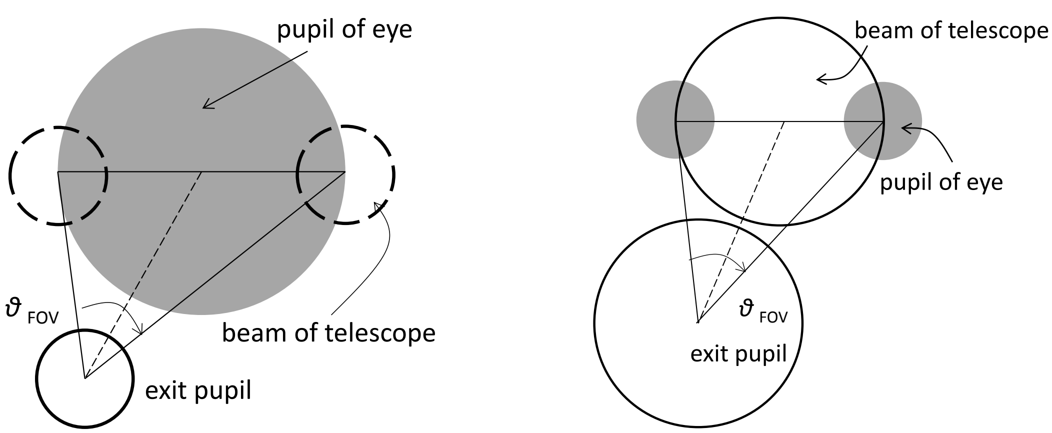
telescopes, it is debilitating. The *Museo Galileo*'s 20x instrument has only a true field-of-view (FOV) of 9 to 21 arcminutes with a 3 to 7 mm iris in the eye. This is only one-third to two-thirds the angular extent of the Moon. In the top of Figure 4, a 111 mm eyepiece is used with about a 1 m objective (9x) and the stereo view has been made about 300mm away from the camera. The bottom Keplerian eyepiece shows the exit cradled in the eye cup, whereas it is deep within the Galilean telescope. Indeed, a young eye must be forced deliberately to focus at infinity rather than on the exit pupil itself.



L parallel R cross L

Figure 4. (top) Three-dimensional simulation of Galilean eyepiece. (bottom) Same for Keplerian eyepiece. If you prefer a cross-eyed view, look at the rightmost pair; a parallel stereoscope-type view is the leftmost pair. Merge the upper or lower edge of the eyecup to become aware of where the exit pupil is floating

The notional geometries depicted in Figure 5 cause the peculiar vignetting diagrams seen in Figure 6. In one pattern the afocal beam bundle slides off the much larger eye pupil, and induces the slope at the edges. The left side of Figure 5 shows the half-on, half-off condition where the illumination would be at 1/2 its 100% value. If the beam bundle were large, this situation would be altered as in the right side of Figure 5 (the circles could be thought of as switching places, with the eye's pupil sweeping across a fat exit beam). Such conditions – common for opera glasses – never take place for Galileo's astronomical



a) Astronomical (high power) telescope b) Low power spyglass

Figure 5. The two illumination patterns leading to each field of view. Imagine the gray circles as being about the same size Vignetting is the correlation of two circles: the beam bundle and the eye's pupil. The foreshortened dotted line is the axis of the instrument.

telescopes. This distinction seems to be the primary difference between the low-power Dutch spyglasses and Galilean astronomical telescopes. The Dutch spyglasses are easy to use by the general public, but the uncomfortable high-power Galilean instruments are of interest only to astronomers.

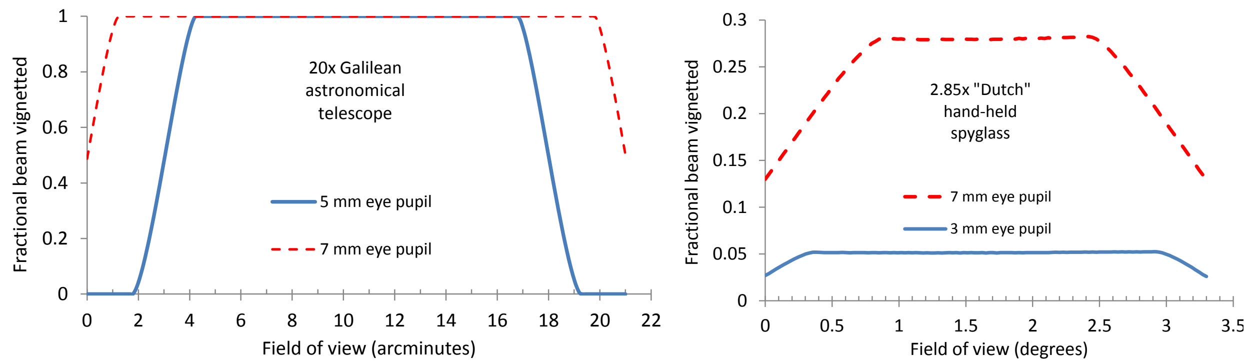


Figure 6. (left) ZEMAX vignetting diagram centered at 10.5 minutes from the axis of Galileo's 20x instrument. FOV is 21 arcminutes for a 7-mm eye pupil and 15 arcminutes for 5 mm. The apparent field is 20x this, or 7 and 5 degrees. This diagram uses 10 mm eye separation from ocular. (right) Similar diagram for low-power spyglasses. Not all of the light appearing at the large exit pupil of the spyglass is intercepted by the eye's pupil, hence the on-axis vignetting.

It is no wonder that many people could not see what Galileo saw when looking through his telescope (and I'm not talking about those that refused to even look).[8] They were attempting to peer through a distant hole with a tiny field lacking a clock drive. It is accepted by modern historians that Galileo was dependent on others in making the telescope named after him, pointing out that most of the invention (in an optical sense) occurred elsewhere.[9] However, the two behaviors exhibited in Figures 5 and 6 show that Galileo's telescope was working in a different area of application. **Note the inversion of the knees of the blue and red curves between figures 6(left) and 6(right)**. This difference may justify regarding the spyglass and astronomical telescope as somewhat distinct inventions.

3.0 MAKING A REFRACTOR SIMILAR TO THAT USED BY GALILEO

I'm not going to claim a *reproduction*. For that, one must enter into considerably more depth than I have time for. Obtaining a reproduction requires duplication of features having nothing to do with optical function.[10] I'm more interested in simulation of optical performance rather than look-and-feel replication. I am going to ask how Galileo's telescopes performed and try – as well as I can with today's uniform glasses and modern surface quality – to simulate the appearance at the eyepiece. I will also consider fabrication difficulties that he encountered.

The important factors in simulating Galileo's telescopes reduce to the following list:

1) focal lengths and apertures of the individual lenses,
2) overall magnification,
3) type of glass (for a similar amount of color dispersion),
4) geometric aberrations of the objective lenses, and
5) material and reflectance of the interior of the tube.

[8] M. Buccianti, M. Camerota, F. Guiduce, *Galileo's Telescope, A European Story*, trans. C. Bolton, p. 93, Harvard Press 2015 (orig. 2012)
[9] *e.g.* Mario Biagioli, "Did Galileo copy the telescope? A 'new' letter by Paolo Sarpi," in *Origins of the Telescope*, pp.203-230, KNAW, 2010.
[10] http://www.scitechantiques.com/Galileo_telescope/

Focal lengths and magnification (characteristics 1 and 2) At the *Museo Galileo* in Florence, Italy, and on their Virtual Museum website there are two complete telescopes and a broken lens originally collected by the Medici family.[11] Of the three objective lenses, only the broken lens seems to have ironclad provenance as being a discovery telescope, since it was mounted in an elaborate display case and is identified as the one that observed what Galileo called the "Medicean Stars" (i.e., the four brightest moons of Jupiter). It would be out of character to believe that Galileo would have delivered incorrect examples to his future patrons. On the other hand, there is no reason to believe that the other telescopes in the museum are the discovery instrument of any of the phenomena. If we take *Nuncius* at face value, the 30x was an early, if not the first, astronomical telescope. The 14x or 20x may or may not have been used by Galileo in his earliest lunar observations, although when and in what order they were made is in some question. He made dozens of lenses by late March 1610, choosing only the best of them to be mounted.[12] Also, the ocular lens on the roughly 20x instrument has been lost in the meantime, and it was replaced with a double convex lens giving 20.6-power.[13,14] Because the eyepiece must be placed inside the focus of the objective by approximately its focal length, the replacement was likely equal to or slightly higher magnification than the original, else the telescope might not have focused after it was substituted.

The on-line *Museo* claims that the shorter instrument was made *circa* 1609 or 1610. The *Museo* also claims that the 14-power instrument was made *circa* 1610. The 14x tube is unadorned, just like a typical laboratory instrument, while the supposedly earlier 20x telescope is covered with gold-tooled and dyed leather. The 14x instrument thus seems to be more primitive. Anyway, authors Bolt and Korey consider the possibility that the 14x may have been made earlier than the 20x.[15]

The optical data of the *Museo Galileo* telescopes are listed in Table 1. The last line in the table lists the isolated lens inside the ivory/brass display case and assumes it is the 30x. Ronchi and Greco *et al.* tested these objectives, and reports they are about perfect within their clear apertures, with the 30x the best (other than being broken). This simplifies the job of simulation considerably. There is no requirement that they be made to some sort of substandard criterion. Note that exit beams are loosely scattered around 1.3 mm, similar to medium to high power for a modern telescope. Galileo seems to like high magnification but not so high that they would cause excessive eye artifacts or image dimness.

Table 1: Parameters measured for the Galileo telescopes and/or objective lens.
From Greco, et al., with right three columns added by me. [16]

Units = mm unless noted	Front	Back	Central	Full	Aperture	Focal	*f*/#	Power	Exit Pupil
	Radius	Radius	Thickness	Diameter	Diameter	Length	unitless	Unitless	Diam
Objective 1	2700	950	2.5	51	26	1330	51	–	
Eyepiece 1	Plane	48.8*	3.0	26	11	-94.0	–	14.1x	1.8
Objective 2	535*	Plane	2.0	37	16	980	61	–	
Eyepiece 2	51.5*	51.5*	1.8	22	16	-47.5	–	20.6x ?	0.8
Objective Lens	940	12000	4.0	58	38	1710	45?	30x?	1.3?

* Greco *et al*. lists these data from Ronchi, V., *L'Universo*, 4, 791-804, 1923.

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11 http://catalogue.museogalileo.it/object/GalileosObjectiveLens.html ;
http://catalogue.museogalileo.it/object/GalileosTelescope.html
http://catalogue.museogalileo.it/object/GalileosTelescope_n01.html

12 M. Buccianti *et al.*, *Galileo's Telescope,* p. 62, 2015 (orig. 2012)

13 Sven Dupré, *Galileo, the Telescope, and the Science of Optics in the Sixteenth Century: A Case Study of Instrumental Practice in Art and Science*, p. 2, PhD Dissertation, Univ. Gent (Ghent), 2002.

14 http://catalogue.museogalileo.it/object/GalileosTelescope_n01.html

15 Marvin Bolt & Michael Korey, "The world's oldest surviving telescopes," in *Origins of the Telescope,* pp.239-241, KNAW, 2010.

16 V. Greco, G. Molesini, and G. Quercioli, "Optical tests of Galileo's lenses," *Nature*, vol 358, 9 Jul 1992.

There are three telescopes to choose from in our modern simulation:

1) +1330 mm used with 14x eyepiece of –94mm focal length. Maximum tube length = 1330–94 = 1236 mm. Max aperture = 26 mm.
2) short (+980 mm) used with a replaced eyepiece of –47.5 mm focal length. Maximum tube length = 932 mm. Max aperture = 16 mm.
3) long (+1710 mm) having a missing tube and eyepiece, presumably giving 30x using a –57mm focal length ocular. Maximum tube length = 1653mm. Max aperture = 38 mm.

As long as the net magnification is approximately reproduced, one would struggle to tell the difference at the eyepiece.[17] Telescope (3) has been made by Gainer,[18] but a 1.6 m tube seems to be a bit difficult to mount, and harder to use (the field is only about 60 percent that of the 20x). On the other hand, it seems to be the instrument specifically mentioned in *Nuncius.* At 0.58 diopters, it is between the focal lengths of off-the-shelf eyeglass lenses, which generally increment in 0.25 diopters. Telescope (1) is easily simulated by a 0.75D ophthalmic lens. I might add that eyeglass lenses are always meniscus (i.e., both sides bulging to the same side) but it makes little difference at such slow aperture ratios. The optical uniformity of ophthalmic plastics is lower than modern glass, but so was the glass of Galileo.[19]

Based on the availability of multiple 1-meter focus lenses to use during testing, I decided my first Galileo-type telescope would be the 20-power with approximately a 980 mm objective lens. I purchased an inexpensive 1 diopter camera close-up lens mounted in a 52-mm filter ring. The 1-diopter lens measured out to a shorter 965 mm, which is 98% of Galileo's, and is a positive meniscus. A few surplus lenses were also purchased.[20] These are in Table 2.

Table 2. Inexpensive simple lenses purchased for trial telescopes

Purpose	Type	Focal length	Diam	Coated
Galilean eyepiece	DCV	–48 mm	25.4 mm	Yes
Galilean eyepiece	PCV	–105 mm	32 mm	No
Galilean eyepiece	DCV	–111 mm	21 mm	Yes
Objectives (10 of these)	PCX	~ +950 mm	50 mm	No

DCV = double concave, PCV = plano-concave, PCX = plano-convex, PMN = positive meniscus

The –48 mm DCV seems to be closest in overall power (20.1x with the 965 mm objective), so it will be used unless it proves unsuitable in other ways. Please note that it is coated, as is the close-up objective, so the telescope will have a little greater throughput. The ten nominal 1000-mm educational lenses were all of shorter focus, with focal lengths roughly 950 ± 10 mm. I also purchased a more expensive 1000.0 mm BK7 lens made to modern professional standards for comparisons during testing.

Glass type (characteristic #3) The remaining characteristic of bubbles and color cast will be ignored. Judging from the front and back radii in Table 1, the lenses are reproduced assuming they are simple soda-lime crown glasses, which will be modeled as Schott B270 crown glass. My close-up lens is probably something slightly different, such as Schott BK7, but adjusting the respective curves so that they match B270's optical power will consign the remaining differences (particularly dispersion) to a minor correction. The educational lenses probably match B270 pretty well. The greatest difference between modern optically transparent lenses and Galileo's are the green or yellow tint of old plate glasses (due to trace minerals), and some bubbles or striation in the older glasses. Galileo wisely chose mirror glass for

[17] The longitudinal chromatic aberration of a given objective lens is proportional to the focal length, and with a 1 mm pupil leaving the telescope, it is not much affected by the eyepiece.

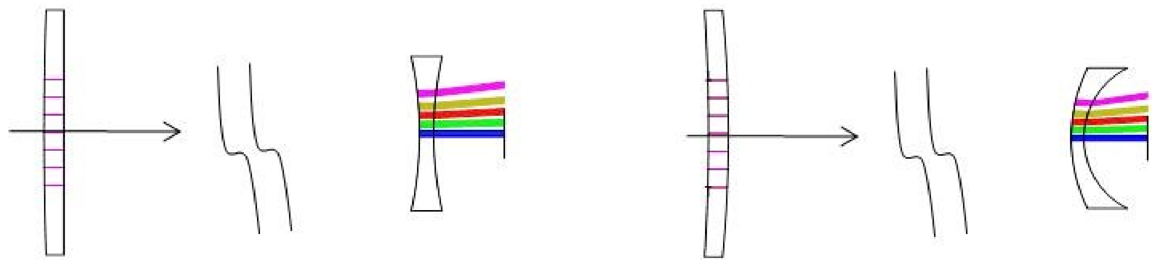
[18] M.K. Gainer, "Construction of a 17th century telescope: An experiment in the history of astronomy," *The Physics Teacher*, vol 19, 22-25 (1981)

[19] M. Valleriani, *Galileo Engineer*, pp. 48-50, Boston Studies in the Philosophy of Science, vol 269, Springer, 2010.

[20] www.surplusshed.com (Long-focus lenses also available from Opto-Sigma, but I have not tried them.)

his lens blanks because it was probably pre-selected to be clearer. The bubbles would cause some scattering, which would act as simple obstructions. The real problem with the primitive glasses would be lack of annealing, especially around flaws. The green or yellow cast actually helps, because green and yellow diminishes the colors forced farthest out of focus by chromatic aberration. The ten 1000 mm educational objective lenses in Table 2 are slightly green too.

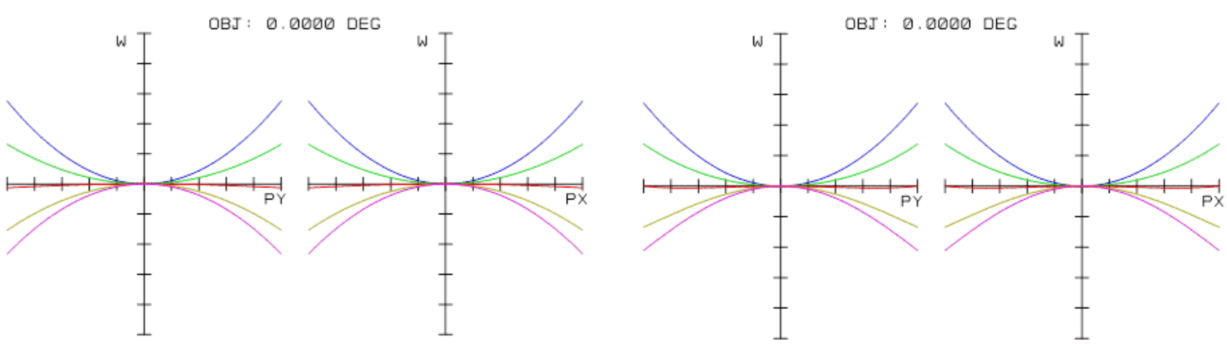
Lens shape (characteristic #4) A lens may have all sorts of shapes (called "bending," as in Figure 7) and yet have the same focusing power. Other aberrations may be varying, in particular spherical aberration. Let's look at the aberration of strongly bent lenses. Please note that lens-bending does not include polished-in errors on the surface. The models are perfect spheres.



a) 20x instrument as described in Greco et al.   b) 20x instrument with severely bent lenses

Figure 7. A severe bend is applied to the same power lens in ZEMAX. The wavy bars mean some length was cut out of the length of the tube in the diagram.

The on-axis wavefront diagrams appear in Figure 8. As can be readily seen, such slow optics with narrow beams of light (16 mm/20 = 0.8 mm) deliver very little difference. The bends of all the lenses used are not this severe.



Left: on-axis wavefront of bend a): "Galileo's"   Right: on-axis wavefront of bend b)

Figure 8. Optical path difference diagrams of two lens-bending situations depicted in Figure 7. Off-axis wasn't much affected either. (Scale = 1 wavelength.)

Low-angle scattering (#5) Even black paper tends to reflect well at low grazing angle. Still, an effort should be made to line the upper end of the telescope's tube with dark paper. Farther down the tube, the visible circle of the lens offers a lower solid angle, and so the scattering is starved of bounced light. The modern designer would block off much of the scattering with at least one annular stop about halfway along the tube, and perhaps two, but this telescope is supposed to simulate what Galileo used, so it will have to be made the old way.

3.1 BUILDING THE TUBE ASSEMBLY

First, a tube had to be found. This is a lot harder than it appeared at first, because one tube must smoothly draw inside another without binding. Luckily, the depth of focus is enormous, so that focusing is not so tender. Using the diffraction depth-of-focus relation appearing in the "Chromatic Aberration" section of the Appendix, the depth of focus is 8 mm (Δn = 1/2 = ±1/4).

As it turns out, a fairly good slide fit between two commonly available tubes was devised. The outer tube is a 2-inch inside-diameter (ID) schedule-40 electrical conduit made of dark gray plastic. The inner tube is a schedule 40 1.5/1.9-inch ID/OD nominal diameter PVC plumbing tube. The felt half of Velcro® strips fill the gap. The tubes nest with smooth friction fit. Just twisting the inner tube as it is pushed or pulled serves to ease withdrawal or advance of focus. Because the tube is short, sag was insignificant, but if an attempt were made to simulate the 1710 mm, the tube would need reinforcement.

Strips of cardboard about the same width as the lens is thick were cut and lightly kerfed with a sharp knife to use as centering arcs. Two annuli of thick dark paper were cut out with scissors and again a sharply pointed knife to provide the aperture stop and backing ring. It prevents the cardboard spacers from falling through. The annulus with the largest hole is placed in the rear of the lens and is not optically active. I could, however, test a wider aperture by removing the front stop. A 160 mm piece of dark paper about 460 mm long was cut out for lining the upper end of the tube.

Before I cut the tube, I had to decide on the longest negative focus I would ever use. Galileo showed a 7 to 8 power to the Venetian senate in 1609,[21] and I wished to simulate it too, even though it most likely had a focal length somewhat less than a meter.[22] A –105 mm eyepiece had been purchased to use for a 9x. For this eyepiece, the tube had to be at or shorter than about 965–105 = 860 mm. However, for it to be focused by a person who doesn't want to wear glasses, the shortest tube correction is typically given with an eye correction of 2 diopters. The effective focal length of the eyepiece (to compensate for the eye power) moves to –133 mm, so 965-133 = 832 mm separates the objective and the eyepiece. I cut the tube where the separation was 820 mm to provide more eyepiece tube to grasp.

Figure 9. Interior of tube after papering upper end. The sloppy fit improves the baffling, so it was tolerated. (Aperture stop and lens not installed yet.)

As soon as the rings and tube were cleaned up with a file and knife, the upper end of tube had to be internally papered. The 460 mm black paper roll appears greatly foreshortened in Figure 9, and it is apparent that at such a grazing angle the paper (and any level surface) doesn't work very well. Only macroscopic roughness would block light efficiently.

Next was assembly of the objective cell. First I inserted a spacer ring in the end section, second the paper annulus with the larger hole, and next the lens with the curved side up. Then I tucked the cardboard centering arcs around the loose lens until it no longer would slide around very much. I placed the 16 mm aperture ring on the lens, followed it with a spacer, and finally held it in with a snap ring. I mounted the lens somewhat inside to keep it from dewing up so quickly. It was similar to the eyepiece mounting in the next section.

If I were Galileo and not too sure of which aperture to use, I would have made the largest aperture rings enclosing both sides of the objective, and then pushed temporary and easily-removed smaller paper apertures on top of the lens. Correspondence seems to show that is exactly what he did with some

21 E. Reeves, "Complete inventions: The mirror and the telescope," in *Orig. of the Telescope*, pp. 167-168, KNAW Press, 2010.

22 Y. Zik, & G. Hon, "History of science and science combined: solving a historical problem in optics—the case of Galileo and his telescope" *Arch. Hist. Exact Sci.*, 20 Jan, 2017, Springer.

telescopes.[23] Both the 14x and the supposedly 30x separate lens – even if made perfectly – have a soft focus, owing to chromatic aberration from a too-large aperture (see the Appendix section on chromatic aberration). Maybe smaller paper annuli were originally associated with these telescopes, and they have long ago been lost in the shuffle of moves, war, deterioration, or changes of custody. Pettit inspected a Cipriani reproduction of the 1330 mm instrument made for G.E. Hale in the early 20th century. He drew a 13/16" = 0.81 inch = 20.6 mm hole in thin cardboard (the marking matches the drawing), yet reported the same 26-mm aperture of Greco in the text.[24] Either this is a peculiar form of typo or Cipriani modeled the reproduction using an alternate diaphragm. Only the finished and decorated 20x telescope seems to have close to the aperture having the same chromatic aberration as is tolerated in modern instruments.

3.2 EYEPIECE TUBE

The eyepieces were much easier to construct as complete lower tube assemblies, called "tailpieces," a fact that must have been obvious also to Galileo. I cut off sections of 1.5 inch schedule-40 PVC pipe and trimmed snap rings for both the 105 mm and the 48 mm, making 9x and 20x telescopes out of the same objective upper. Figure 10 depicts a snap ring being cut out of 1.5 inch PVC plumbing. Another reason that made assembling a separate tailpiece for each eyepiece preferable was that the tube was much longer for the higher power eyepiece. Recall that the main tube must be shortened beyond the entire focal length of the lowest magnification eyepiece. The higher power eyepieces are farther away from the objective – in this case by about 105 – 48 = 57 mm.

Figure 10. Assembly of a "lower" tailpiece or eyepiece end. The "upper" end is similar and not shown.

4.0 PERFORMANCE

I mounted the instrument on a temporary camera-tripod mounting. It was not adjustable in tiny increments, but could be used to look at fixed terrestrial targets. Lacking a bell tower or beautiful Renaissance skyline as a target, I used a treetop and a power line at a range of about 150 m against the

[23] Dupré, 2002, pp. 286-289.
[24] E. Pettit, "A Telescope of Galileo," *Pub. Ast. Soc. of the Pacific*, *Vol. 51*, p. 148 (drawing) 1939.

daylight sky toward evening. Field of view with roughly a 3-mm opening in my eye was calculated to be 9 to 11 arcmin, so the telescope was challenging to point. It seemed to work acceptably (but not well) at 9 power using 105 mm. Then I tried the 48-mm eyepiece. The first thing I noticed was that it seemed to be in much the same focus over about an inch, instead of the 8-mm region calculated previously. Lack of crispness on the focus indicates poor optical quality. Another difficulty was that a precisely centered target seemed to soften, but it appeared to be in better focus at the edge of the field. I suspected that the 1-diopter camera-filter lens had a zone at the center. The crescent Moon that night displayed a feathery look at the crescent points and a blurred appearance at center.

The first mechanical problem is that the instrument was nearly impossible to point and move with this fixed mounting. The photo tripod was unbalanced and clamped into place. If the target was by chance within sight, a typical field-of-view of 15 arcminutes quickly moved aside in little more than a minute (arcminutes on the sky translate to clock minutes in about a 15:1 ratio at ecliptic declinations). Even though Galileo emphasized the requirement that the astronomical telescope be fixed in position and operated without holding it, it was also apparent that he required an unclamped, balanced mounting that is adjustable in tiny increments.

One might hope to obtain information by examining old pictures of telescope mountings in use, but graphics about early astronomers are not very realistic. The artist often shows the user peering out the window with a shaky little stand that is seldom pointed more than five degrees above the horizon. Often, these were painted long after the scientist's death by non-astronomers. A good example is a fanciful picture of Galileo where the artist shows him using the old Medici museum holder made decades after his death, even including the ivory and brass plaque for the broken lens!

One exception to this trend is the "Man with telescope" from Helvelius's *Selenographia*,[25] which shows an early mounting used out-of-doors at a significant angle. The drawing is reproduced in Figure 11, and features useful items likely copied from actual instruments. It appears to be a trunnion-type altazimuth mount of adjustable height. The off-center telescope apparently is counterbalanced by a slightly thicker board running parallel to the optical axis on the other side of the trunnion bearing. Various appliances that can be traded with the ones in use are shown scattered about the base. It has musical-instrument style adjustment pegs everywhere and an altitude measurement tool. This drawing was clearly based on a real instrument.

I made an offset fork mounting based on this one (only using modern materials) It is shown also in Figure 11. It looks superficially like a Dobsonian mounting, but the trunnion does not use PTFE as a bearing material and the azimuth bearing is an axle rotation in the photo tripod.

The telescope also badly needed a finder. Tycho Brahe had balanced alidade sights capable of directing instruments to a fraction of an arcminute, so this was a known technology, albeit difficult to use without motor drives or limits like transit observing. It demanded great skill and advanced error reduction techniques.[26] It also worked best for bright targets. Galileo mostly had bright targets, but he also looked at star fields.

If one were to look through the eyepiece of a straight-through refractor (like a finder or binocular) with both eyes open and move the center of the field of view to the far-away location of the target as viewed in the unaided eye, the object (in principle) pops into view. For this to work a number of conditions must be met, but the most important is that something must mark the center of the field of view.

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25 Hevelius, Johannes, 1611-1687. "*Selenographia*, man with telescope." (1647) Rice University: found at http://hdl.handle.net/1911/78810 (This is not Helvelius himself, but a stylized depiction of earlier astronomers.)

26 The star or planet is inside one arcminute for only 4 seconds in time, and Tycho had no accurate clock or clock drive. Thus, measuring the angular separation of two objects within 1 arcmin was a stunning achievement.

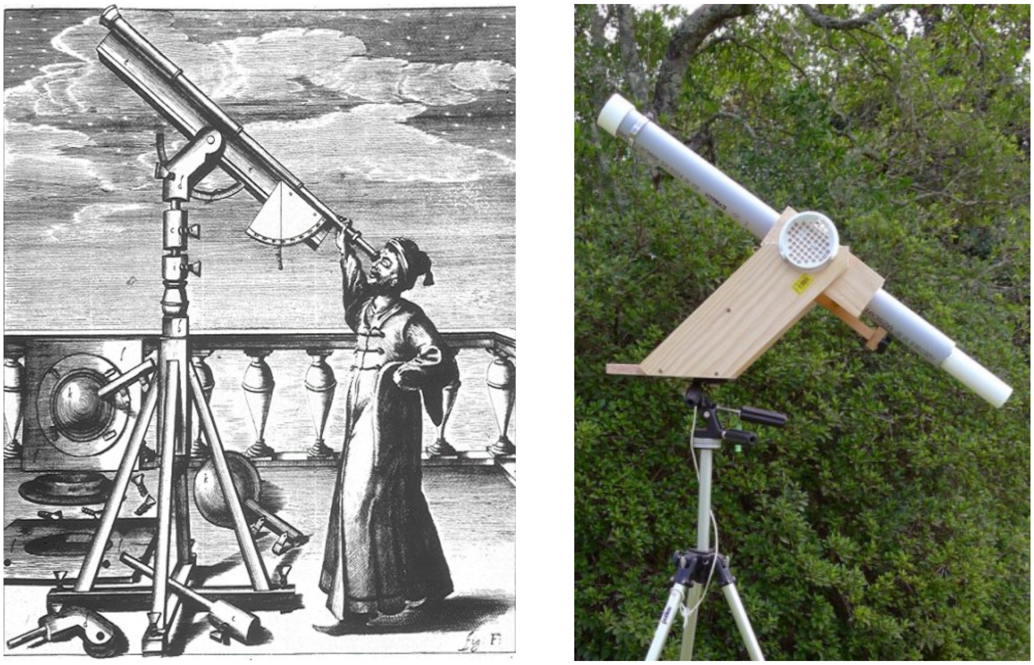

Figure 11. "Man with telescope," from *Selenographia* (1647) and my simulation of the 20x/980mm telescope. Note appliances around base, including height extension sitting athwart the base crossing.

The exit pupil was small and the direction could be estimated this way. But the exit pupil was too dim. I could barely see it and the field of view was not sufficient to allow an error. I would sweep back and forth hunting for Jupiter, only to have it flash through the field during the search. By the time I halted my sweep, it was gone. A lower power finder or a set of adjustable peep sights was needed. One could envision a white-painted circle at the objective end of the tube through which a tiny hole at the rear serves as the peep sight. A dim candle held in a lantern several meters behind the observer would illuminate this circle well enough to make it usable without completely ruining night vision. I ended up (after some false starts) with the finder at the right in Figure 12. I illuminated it handheld with a red LED taking the place of the candle.

The instrument also suffered from lack of a star diagonal. We now rely automatically on such devices without considering what visual observers would do before they were invented. Look at the photo in Figure 11 and imagine the tube kicked up to 70 or 80 degrees altitude. The trunnion rotation axis must be a great deal higher or else the observer is forced into an uncomfortable half-crouch. The trunnion ought to be at least 2.5 meters high to be used standing up, since climbing a small step for lower objects would be easier than spending the whole observing session bent over. Sitting down creates other issues with reaching the eyepiece and cocking the head back at an unnatural angle.

Since the original 1-diopter objective performed so badly, it was replaced with the best of the "educational" lenses (described below in the testing section). The Moon was seen better; I observed the giant terminator crater Langrenus on a 2.5-day moon at roughly 9x using a –111 mm tailpiece and saw half of Mare Crisium lit from the side. It looked a great deal like Galileo's drawings with the terminator a little farther in. The 20x view of the Moon was good as well. For both eyepieces, the moon tended to slide quickly out of the field.

Turning to Jupiter, I could not locate any of the four brightest moons. My view of the planet was clear enough. So where had the moons gone? Because of moonlight, I estimated that my unaided view of stars was limited to about magnitude 3.5 or 4, but that underestimates telescopic limits with a small exit beam.

Figure 12. (left) Velcro felt tabs on withdrawn lower tube that allow smooth focusing. Insert tube to felt-tip mark to achieve distant focus. (right) Finder is a set of cup rings.

The amount of brightening I could expect was, assuming a pupil of 7 mm, $(16\text{ mm}/7\text{ mm})^2 \sim 5.2$ or a little under 2 magnitudes. The brightness of Ganymede near the time I was viewing is around magnitude 5.4, so I should have seen at least one moon increased by the light-gathering power to about magnitude 3.5, with the rest trailing about 1/2 magnitude behind. Changing telescope to a small Newtonian with most of the aperture blocked by my hand, I could see one of them with a modern 28-mm (15 power) eyepiece. Later in the year, when Jupiter was moving closer to solar conjunction, Ganymede dropped to magnitude 6.1, so with the 20x telescope it was a little over 4th magnitude. Ganymede still wasn't visible. There was no Moon at this time, but haze still constrained the detectable limit. Galileo may have been using averted vision (direct-vision thresholds are about three magnitudes brighter than averted thresholds[27]), but this makes estimating planet-moon separations problematical.[28]

Next was Saturn and, because I knew what I was looking at, I thought I could see the rings. Yet I have to admit that if I had looked at Saturn with unknowing eyes, I would have seen the planet only as a flat oval. This is so easily confused with an out-of-focus astigmatic point source (all too common with such crude optics) that I conclude that Saturn's unusual appearance was discovered with the larger aperture and higher magnification of the 30x.

This experience somewhat matches with that of Ringwood, who found 20x inadequate to explain the views of Jupiter and Saturn, even with the larger aperture of the 1710 mm telescope, and contradicts Drake, who believed that it was simple with the 20x.[29,30] I found that it takes a larger aperture than 16 mm to easily see what Galileo saw at Jupiter. On the other hand, the limited field of view of the 1710 mm (see Table A-2 in the Appendix) makes the whole linear spread of the satellites too large to encompass at the same time. Maybe he viewed only one side at a time. Based on the behavior of the apertures and magnifications I used, I infer the 20x telescope in the *Museo* is not the one used for the Jupiter observations.

4.1 ABERRATIONS

I calculated the ideal apertures of Galileo's three telescopes, assuming nothing was wrong with them other than chromatic aberration, and that the modern tolerance for simple achromats applied (i.e., $\Delta n = 1$, or red

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27 Chris Lord, "An investigation into the photopic threshold limit in a telescope," www.brayebrookobservatory.org/BrayObsWebSite/HOMEPAGE/forum/PhotopicTelLim/PhotopicTelescopicLimit.html

28 The next opposition, I saw the moons through a 23-mm aperture with slightly averted vision.

29 S.D. Ringwood, "A Galilean Telescope," *Quar. J. R. Astr. Soc., vol. 35*, pp. 43-50, 1994.

30 S. Drake, "Galileo's First Telescopic Observations," *Journal for the History of Astronomy, vol vii*, pp. 153-168 (1976)

and blue wavelengths 1 wave out-of-focus, explained in more detail in the Appendix). The result is the solid red line in Figure 13, which is safely inside the diffraction-limited region with weighted Strehl ratio greater than 0.8.[31] However, Galileo was fighting other problems besides spectral color. His 20x instrument is set neatly on this line, but the 14x and 30x instruments are both too large for the Δn =1 condition. Let's speculate that the desire for light-gathering power overrode the need for good definition in the longer instruments or perhaps he had additional smaller paper annuli that are now lost. (A simple model for additional magnitudes is at the right based on the aperture listed on the left vertical axis.)

Before I limited the telescope's aperture to the Δn =1 line, I decided to try the Δn =2 condition. For the 20x, the 16-mm opening was replaced by a 23-mm aperture. Results were surprising. I was not bothered by excess color. At 9x, the moon looked much like it does in a good pair of binoculars (with a smaller field). I could see large craters and mountains near the terminator. Even at 20x, I saw a Moon which did not display greatly objectionable color error. I believe the error induced by a decentered eyeball chasing the rapidly changing scene was worse. Field of view was still less than the diameter of the Moon. We do not know and Galileo only boasts about his maximum magnification in the *Nuncius*, but I believe he did most of his lunar observing at 20x and below, agreeing with the opinion of Drake previously cited. A magnification of 30x is not necessary for the lunar pictures that appeared in the *Nuncius*, and using the more limited field of view is more difficult, although the extra-large crater at the center may be caused by his choosing higher magnification to get a better look at that specific object. It seems the achromatism condition can be stretched somewhat if one needs it. Galileo very much needed it.

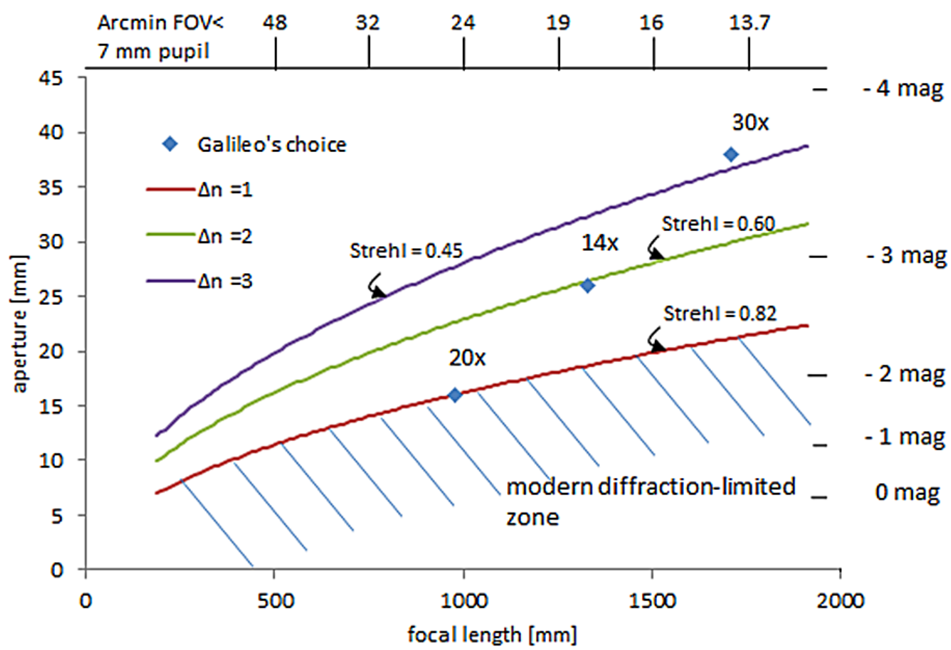

Figure 13. Aperture vs. focal length chart for simple-lens Galilean telescopes. Better than Δn=1 applies to modern refractors. Brightness increase in stellar magnitudes re 7 mm pupil at right. Along the top are calculated the maximum fields-of-view. Real FOVs are smaller because the observer draws back from the eyepiece

4.2 OTHER OBSERVATIONS

Galileo also looked at the stars, although this result of *Nuncius* is the least discussed. It seems that these data may have been added right before publication. Of the stellar portraits, the only asterism rendered with anything approaching successful recognition was that of the Pleiades (not shown), probably because

[31] Born M. and E. Wolf, *Principles of Optics, 7th ed.*, pg. 528, Cambridge Press 1999.

of the plenitude of bright reference stars. One can identify the broken-line star chain having six members immediately to the south of Alcyone, the farthest away of which is listed at magnitude just under 9. Again, this argues for the over 3 magnitude reach of the aperture used in the 30x, but at least it demands the 26 mm of the 14x. The 16-mm aperture of the 20x would have elevated only some of these minor stars to a marginally detectable brightness. They might be possible to observe at the threshold of vision if you already know that they are there, but difficult to discover without some excess brightness. With averted vision, I saw the first three stars in the chain easily (about 7th magnitude) and the 4th and 5th star in the chain only intermittently with a 16-mm aperture covering my *f*/4 reflector (magnitude just under 8), whereas with a 37-mm aperture, even the 6th stood out clearly. With these apertures over a Newtonian reflector, I was doing a best-case example. It would only be worse in Galileo's simple refractor.

Galileo used the term "Orion Nebula" for a different feature, a fuzzy glow of a sparse cluster around the neck or head of Orion. We no longer think of it as a nebula, but it is no surprise that people saw the night sky a different way in those pre-telescopic days. Figure 14 shows *Nuncius* originals of what he called the Orion and Beehive nebulae flanked by some *Stellarium* diagrams.[32] I have underlined where in the text Galileo refers to "Orion's Head" and marked a suspected correspondence between the diagrams using brackets. If we were to look only at the *Nuncius* figure and go to a diagram around the modern Great Nebula in the sword of Orion, we would be very confused in trying to identify anything.

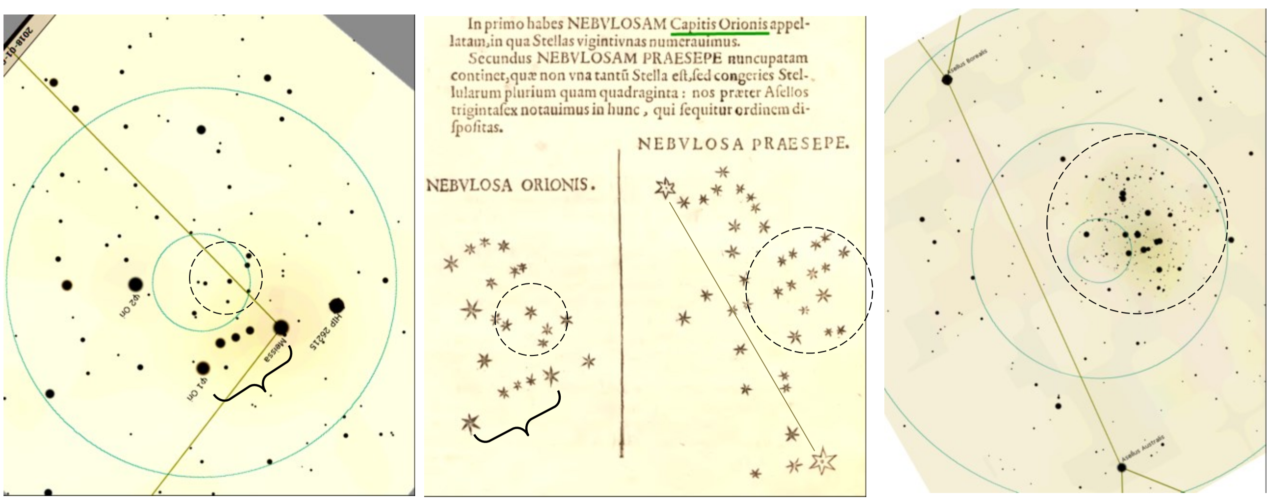

Figure 14. *Stellarium* diagrams outside of *Sidereus Nuncius*. Green line underlines "Orion's head." [33]

Galileo did a fairly good job of hopping from star to star when close strings existed (as in the bracketed sequence of Orion's Head or the chain of dim stars below the Pleiades), but when they didn't form a string (as in the Beehive) he was only guessing the position. The reason for this is, of course, the limited field of view. At the center of the two *Stellarium* diagrams is the 30-arcminute circle of a standard Telrad™ finder. Using the *Museo*'s 20x or 30x, he had a maximum field of view of 21 and 12 arcminutes respectively (see Tables A-1 and A-2 in the Appendix). That is about 1/3 to 2/3 of the smallest Telrad circle. If I have oriented the Praesepe figure correctly, he identified the west-of-center clustering with respect to the line between the Aselli stars in Cancer, and that is about all.

Incidentally, a little bunching of stars in the Orion's Head "nebula" is circled. Position information is inadequate to identify individual stars, but if they correspond to the stars indicated in *Nuncius*, they are all between magnitude 8.6 and 9.3, which again argues for more aperture than 16 mm.

[32] *Stellarium 0.14.2* (planetarium simulator), www.stellarium.org, 2000-2016.

[33] In *Nuncius*, some star figures are rotated oddly, I assume for the convenience of the printer.

5.0 SPECULATIONS ABOUT TESTING AND FABRICATION

I decided to test this objective with a Ronchi screen. This is an array of linear black and white bars placed near focus, acting somewhat like a set of side-by-side knife edges in the Foucault test.[34] The test was in double pass auto-collimation mode against an optical flat that is smoothly better than 1/4-wavelength, implying that its refocused accuracy is much higher. It is diagrammed at the top in Figure 15. At the lower left, the modern 1000.0 mm professional lens shows the expected appearance of the Ronchi test on a good lens (and incidentally demonstrates the flat mirror is good). It is taken with a 75 lines/in (37.5 line-pairs/in or 1.5 lp/mm) Ronchi ruling.[35]

How did the 1-diopter positive meniscus close-up lens perform in this test? In Figure 15 at the right, the lens shows an unusual kink near the center and tilted lines, indicating astigmatism and a central zone. Other focus positions show this as a deep concavity at the center. Note that this picture is taken with an even milder Ronchi ruling of 1 line-pair/mm. It is quite smooth, though.

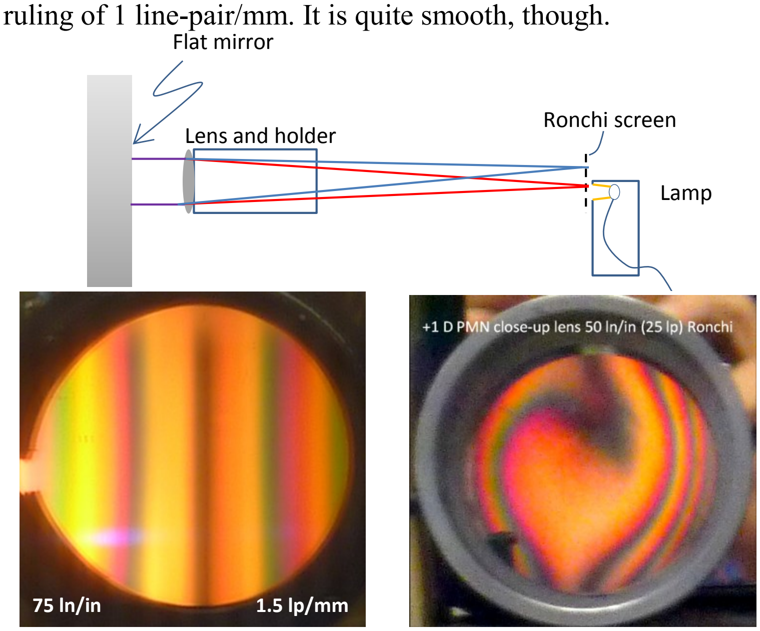

Figure 15. Double-pass auto-collimation Ronchi test and pattern of 1000.0 commercial lens and 1D close-up lens.

I also inspected 10 inexpensive "educational" lenses in this manner.[36] Each of them had equally bad central zones or severe roughness except for one or two. This best educational lens shows a small amount of spherical aberration, some astigmatism, and is a little rough. But it was clearly better than the others and probably equaled an acceptable lens that Galileo would have encountered in his efforts. The fractions of successful lenses were approximately the same too. He stated that by March of 1610, he had made about 60 lenses, only a few worth using.[37] These modern educational lenses show a satisfying green tinge of simple plate glass, similar to Galileo's.

The Ronchi test for some of the other lenses is depicted in Figure 16. They show random or radial roughness, holes at the center, and the yin-yang appearance of astigmatism. Galileo's problem must have

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[34] A. Cornejo-Rodriguez, ch. 9, "Ronchi Test" in *Optical Shop Testing 3rd ed.*, ed. by D. Malacara, Wiley 2007.

[35] See the Appendix for a numerical way of interpreting the Ronchi test.

[36] "Inexpensive" doesn't mean that these lenses are not useful. It will be shown below that we can learn a lot from them, because having imperfect lenses was a problem Galileo had to confront, too.

[37] M. Buccianti, *et al.*, *Galileo's Telescope, A European Story*, p. 62, 2015 (quote from *Works*)

been similar. Galileo himself (or more likely, helpers) made many such lenses and only a few were useful. How did Galileo sort them without auxiliary testing tools? It was an excessive amount of work and expense to rig up a new tube for each objective, each having a slightly different focal length, only to throw it out after one night's trial.

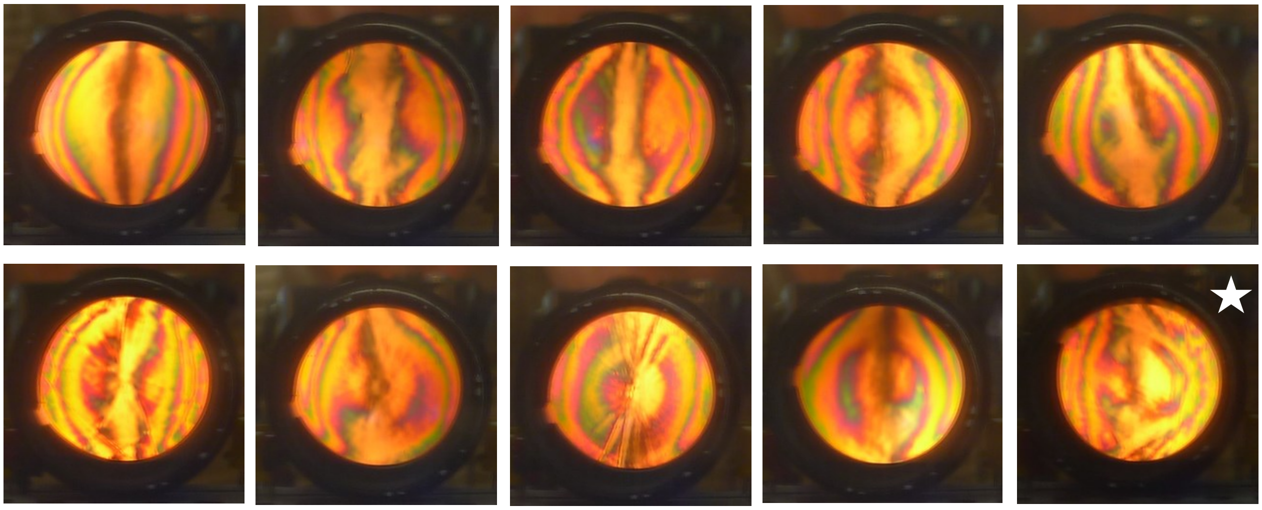

Figure 16. Double-pass Ronchi-test pictures of ten 1-meter educational lenses. Grating 1.5 lp/mm. Only the one in the instrument (shown at upper left) is good. The one immediately to its right is perhaps acceptable.

Galileo first would have rigged an adjustable lens holder, basically a wooden optical bench with a rail along which the separation is adjustable. Not only is this tool useful to discover the precise value of lens separation before a tube is cut, but to test the telescope's overall performance and net magnification. He would either take this "telescope-on-a-stick" out at night or arrange some sort of daytime view at a great distance. The tiny beam exiting the telescope forces such an optical bench test to become a binary judgement: either the lens is good enough or it is not. An aperture mask may have been tried, but not deliberately designed unless additional testing information is available.

Galileo apparently also figured out a method that might yield information about not only the magnitude of optical errors but their position. Such lab testing must have happened before he could make the aperture mask oval, as mentioned in a letter to a member of his future patron's house in January 1610.[38] A non-circular mask involves the adjustment of three variables (size of aperture, shape of opening, and axis orientation), even if it is centered. Assuming three values of each, this is 27 mask trials to cover all the options. Such a search cannot be explained by direct observation of the subtle differences in the final images. It must be an optical-shop test.

Galileo, in lab-checking his lenses, would have needed a way of tossing them into one of three bins: 1) lenses justifying further tests with the focusing rail, 2) lenses that were not good enough at the time but that might respond to a return to the fabrication process (or perhaps just needed a mask), and 3) rejects.

What testing method was Galileo capable of and likely for him to use? It must have the following characteristics:

A) The test couldn't have been a long stretch from the common purposes of a single lens. The optical theory he knew was primitive and mostly confined to use as eyeglasses (if lenses are weak), and hand magnifiers (if lenses were strong).

[38] A. Van Helden, "Galileo and the Telescope," in *Origins of the Telescope,* p. 192, KNAW, 2010.

B) The test wouldn't have had much applicability for longer and larger Keplerian telescopes, or else it would have worked its way eventually into the canon of telescope-making techniques, even if Galileo chose not to report it.

The most straightforward answer to characteristic 'A' is some sort of application as a magnifier. The first thing he would have thought of was magnifying fine details of a printed page or perhaps woven cloth, a common application of a hand magnifier, then as now. In Figure 17, the image of a two-color oxford fabric is shown with a common magnifier held at arm's length. The lens is set slightly inside the "point of inversion" of early optics.[39] One need only assume that Galileo was an excellent observer (a conservative hypothesis, if not a gross understatement) to speculate that he would notice that this configuration is not only a greatly magnified image of cloth *but a test of the lens itself*.

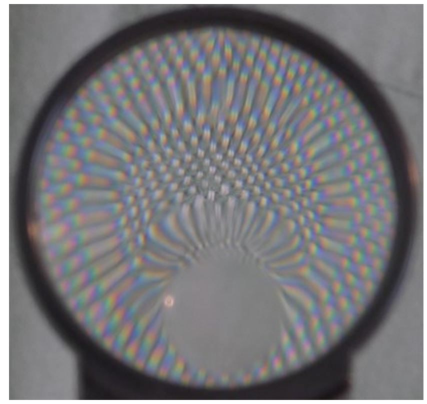

Figure 17. Cloth viewed with an inexpensive magnifying glass

At the bottom of the lens, a higher power lens has been inset into the larger circle and has stressed the glass in the larger piece. Galileo had no inset lens, but he may have had bubbles, grinding stresses, or stones. In the larger area is an annulus where the image of the cloth is blurred radially, and then it improves again toward the rim of the lens. Upon noticing this or a similar phenomenon, Galileo might have tried to set up the same situation with his objective lens, but it would have been at longer distances for his lower curvature lenses.

I conjecture that by fiddling around with the configuration, eventually Galileo set the trial telescope lens at about the infinity focal distance away from a printed-page target. He crossed the room and turned back toward the page. He would see an encouragingly large letter. Let us say he went back to the lens and moved it a few centimeters. The letter changed in size, but still not enough. By going iteratively back and forth to the viewing position, he eventually came up with a compromise lens position somewhat beyond the position of infinity focus, the precise value depending on viewing distance. He withdrew across the room a last time (say 3 to 4 meters for the 1 m lens) and saw large letters. This seems to be what Porta was describing in a prior work, and all that Galileo had to add to it was to use such a configuration as a test of a lens.[40]

Now let us infer what is happening optically in Figure 18. At some distance outside of infinity focus the lens would, in principle, project a tiny portion of a letter right on the pupil of the eye. The lens is at the "point of inversion" in terms Galileo would understand. The letter would appear completely defocused. If the optician recovers an image by moving the lens or the observer a bit closer to or farther from the target, the letters become smaller and come into focus. If $f_{\text{conjugate}}$ is far enough either behind or in front of the eye, sufficient eye accommodation power allows an image to be brought to sharpness on the retina.

In Figure 18, the eye is replaced by a small camera, and biological accommodation is replaced by manual focus adjustment of the camera. In the configuration shown, it works better to have some more in-travel beyond the infinity focus on the camera, but if such travel doesn't exist, one may withdraw beyond $f_{conjugate}$ to focus on the beam diverging from the real image at $f_{conjugate}$. Here the camera or eye lens has become an

[39] Y. Zik & G. Hon, "Galileo's Knowledge of Optics and the Functioning of the Telescope ─ Revised," p. 7, April 2014, Arxiv.org, 1307.4963

[40] J.B. Porta (GB. della Porta), *Natural Magick*, Book 17, Chap 10, p. 368, 1658, pub. by Young and Speed, English translation of 1558 original having several later editions.

integral part of the test system. It resembles a long, low-power microscope with the strongest performance burden on the tested objective lens.

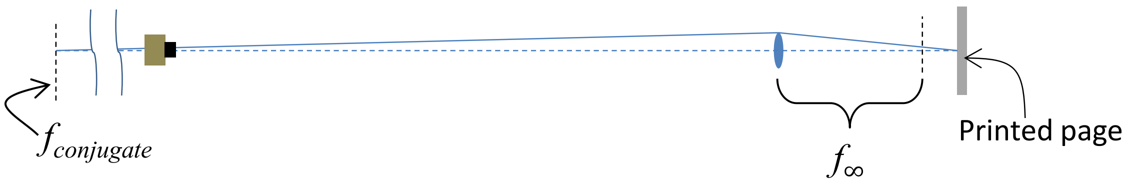

Figure 18. One optical configuration of primitive test.

Figure 19(a) shows the original 1-diopter filter ring close-up lens in such a test. Its astigmatism and central hole are revealed by the radial streaks and fuzziness away from the center. The tester does not have to know this terminology, but it is apparent that the clear aperture is not large, whatever the name. It would probably have worked if the aperture was stopped to 10 or 12 mm, but such a miniature aperture is even smaller than the *Museo*'s 20x. Figure 19(b) is the starred Ronchi test of Fig. 16. It exhibits a fuzzy out-of-focus ring in a narrow radial zone. The radius of this zone is unfortunate; it is inside any reasonable size aperture mask. Figure 19(c) is the educational lens I used in my instrument. The outside black ring is just the aperture mask, and the gray ring is the end of the telescope tube (I did not remove it from the upper tube assembly). Even though it is used at only a magnification of 8, it is startlingly better than the other educational lens, and justified cutting a tube and testing on the sky. Figure 19(d) is a commercial 1000.0 mm BK7 lens made with modern methods and taken from beyond $f_{conjugate}$ (hence the image is reversed). The more obvious spectrum might be due to the wider bandwidth it permits. Sometimes, a green or yellow color is not a disadvantage. The green tint of the educational lenses in (b) and (c) is shown in the frame of Fig 19(e).

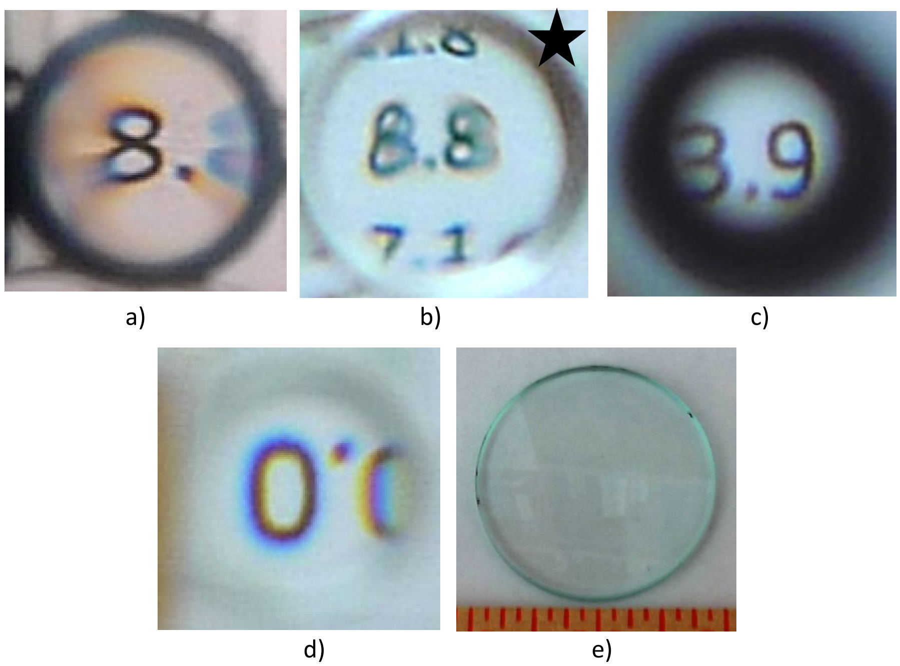

Figure 19. a) Original rejected 1-diopter lens. b) One of the rejected educational lenses in Figure 16. c) The lens used on the instrument d) a modern wideband transparent lens, on the other side of $f_{conjugate}$, e) picture of educational lens in b) and c). Note green color, like common plate glass. Estimated magnifications in a) to d) are about 7, 7, 8, and 11. The picture in (a) is redder than the others because the room was lit by tungsten light.

The test is enhanced by motion of one's head slightly sideways to the optical path. The blurry zonal ring of Figure 19(b) appears fixed as the letters rapidly move beneath it, so it is even more obvious where the lens is good and where it is not.

The lens arrangement should expand the letters to about the magnification of the telescope to allow critical inspection. The letters would fill the lens to magnify so much. Nevertheless, even the fairly mild magnifications of 7 to 11 witnessed here allow the tester to reject the obviously bad lenses and send ones likely to be better on to final checks on the sky. More to the point, it allows an estimate of the size and shape of any aperture mask to be made.

The other demand is that the test must not have moved much beyond Galileo. Very little was reported about methods of testing in the 17th century. Girolamo Sirtori, although he was forthcoming on the technical details of his own polishing and grinding, wrote very little about testing.[41] Only Isaac Beeckman gave much detail about methods of testing, and those seemed to work best on non-astronomical optics. For example, Beeckman was the first to cover the center of a lens to see if spherical aberration was excessive and to effectively describe astigmatism.[42]

Even if it had been reported, this test would not have been of much use to succeeding generations of telescope makers. Later astronomical telescopes were much longer and operated at higher power. The operator would have had to withdraw to enormous distances and expand the letters well beyond the aperture to gain sufficient sensitivity. This test might give informative results up to magnifications of 30 or so, and then on smaller targets than letters, but beyond that it was not applicable.

5.1 LIKELY GRINDING TOOLS

The objectives of all but the first two of Galileo's telescopes were far too long to be of any use in spectacle-making. People might have demanded one in order to hold a page precisely at arm's length, but the easier way of reading is just to hold a stronger lens closer to the face. In any case, methods of measuring weak vision were not yet that precise. But Galileo made long focus lenses that were essentially perfect over what was then a large surface area. He clearly was advancing shop technique.

What were the tools used to make the objective? Galileo is known to have prepared a late November 1609 shopping list that listed abrasive or polishing powders, iron artillery balls, and bowls, presumably as tools for his lenses.[43] Something like these bowls would have been standard optical-shop practice for making positive lenses at the time, with the sides of the bowl cut away and just the mildly concave bottom exposed.[44] The lens grinder would put abrasive slurry in the dish and moved the flat lens blank around until it acquired a more-or-less spherical shape. Possibly the tools were spun on a machine very much resembling a potter's wheel.[45]

Look again at the shortest surface radii of the 20x and 30x in Table 1. One is about 535 mm and the other is about 940 mm. No doubt Galileo had seen bowls in lens-grinding shops, but those would have been most commonly of half the radius he needed. A 200 to 250 mm radius tool gives a plano-convex focal length of focal length 0.4 to 0.5 m. This is a good range to combat presbyopia, the commonest use of eyeglasses at the time, but it is the wrong focal length for Galileo. Also, it would have taken extra effort

41 G. Molesini, "Testing telescope optics of seventeenth-century Italy," in *Origins of the Telescope,* pp. 272-280, KNAW, 2010.

42 Fokko Jan Dijksterhuisin, "Labour on lenses: Isaac Beeckman's notes on lens making," *Origins of the Telescope,* pp. 257-270, KNAW, 2010.

43 Buccianti, *et al.*, *Galileo's Telescope,* pp. 59-60, 2015

44 quote from (della) Porta's *Natural Magick* appearing in F. Twyman, *Prisms and Lens Making*, 2nd Ed., p. 11, Hilger 1988 (orig 1952)

45 Valleriani*, Galileo Engineer,* pp. 45-47 (2010).

to condition new tools for glass work. One could not just buy any bowl and start grinding with any hope of optical quality. A great deal of smoothing and surface regularization needs to be done on the tools before they approached surfaces of revolution. This would be done with abrasive stones or smaller iron plates and loose abrasive. He may have used a crude lathe with a radius tool, although turning such shallow surfaces might be difficult when spun on 17th century plain bearings.

Also consider the unusual production schedule for Galileo's telescopes. When the shopping list was made it was late in November 1609. He began observing the Moon in early December and he moved on to Jupiter in early January. Well into March, he (or his assistants) had made 60 objectives, only a few of which were good.[46] He published *Nuncius* in early March 1610, and it took a finite time to prepare. It seems that early in the reported period of telescope making, he made his discovery instrument or instruments (presumably including the 30x). This is phenomenal speed and success, even taking into account that working small pieces of soft plate glass for refractors is a great deal faster than the large Pyrex glass disks ground by modern reflector makers. (This rapidity is partly offset by the softer abrasives then in use.) Considering also that he had to finish tubes and rig up some sort of mounting, these dates are astonishing.

Galileo was likely using new techniques to fabricate as well as to test them. We cannot know what method he used, but it is clear he made very long-focus, accurately spherical lenses very early in this period. He had to make them quickly to have them ready for the observations he made in the next couple months. The difficulty of making weak lenses is such that he had to understand their advantages to justify pursuing this goal.

It is also possible to estimate the missing ocular focal lengths on the 20x and 30x instruments by noting that Galileo might have used artillery balls as iron tools in forming the concavity. By examining the likely diameters of common 16th century cannonballs, one can infer the radii of the Galilean eyepiece and hence the focal length, assuming the index of refraction is about 1.52. In Table A-2 is a list of cannonball diameters as made by British cannon works prior to later standardization. I have been unable to find information about Italian artillery. Cannon bores were by no means standardized or made to the same diameters in different countries in the 16th century, so you should regard the following as a range rather than a measurement. On the other hand, some effort was applied to making popular types as similar as was possible in that pre-industrial age because expensive cannons were sometimes captured in battle and reused. The yellow ranges are likely ranges of diameters for the 20 and 30-power instruments. The cyan

Table A-2. 16th century English gun ammunition diameters.[47]
ZEMAX calculation is for Schott B270 and the separation is focused on the 70% zone of 555 nm.

Gun Type	Round Shot Diameter (*in*)	Shot Weight (*lb*)	Cannon Bore (*in*)	ZEMAX lens parameters DCV
Saker	$3^1/_4$	$4^3/_4$	$3^1/_2$	
	$3^1/_2$	6	$3^3/_4$	(23.3x 980mm, sep. 934.7 mm)
	$3^3/_4$	$7^1/_2$	4	(21.8x 980 mm, sep. 931.7 mm)
Demi Culverin	4	9	$4^1/_4$	(20.4x 980 mm, sep. 928.7 mm)
	$4^1/_4$	$10^3/_4$	$4^1/_2$	(19.2x 980 mm, sep. 925.6 mm)
	$4^1/_2$	$12^3/_4$	$4^3/_4$	(31.6x 1710mm, sep. 1651.9 m)
Culverin	$4^3/_4$	15	5	(29.9x 1710mm, sep. 1648.9 mm)
	5	$17^1/_2$	$5^1/_4$	(28.3x 1710mm, sep. 1645.7 mm)
	$5^1/_4$	20	$5^1/_2$	

[46] Buccianti, *et al*, *Galileo's Telescope,* p. 62, 2015
[47] "16th century gun calibres," at http://www.arc.id.au/Cannonballs.html

backgrounds mark the precursors of the common 6, 9, 12, and 18-pounders that came to be used in European navies in the next century, and Venice at the time was very much a maritime power.

The most likely eyepieces are based on the 9-pounder demi-culverin for the 20x and the 12 pounder for the 30x. Powers are about 20.4 for the 980 mm objective and about 31.6 for the 1710 mm objective. Thus, the expected high-power eyepieces are double concave with the Table A-2 ZEMAX-modeled magnification ranges. Separations are between vertex centers of finite lenses (4~5 mm thick). The plano-concave eyepiece of the 14x telescope could be generated from an uncommon 7.5-pound saker, but was more likely a purchased eyeglass.

It has also been suggested the original eyepiece lens of the 20x telescope was a plano-concave and the replacement a convenient double concave pulled from the shelf.[48] The hypothesis is that Galileo would have had to take more time to make a double concave lens and he was rushed. However, ZEMAX modeling of this lens calculates that a plano-concave lens of the same power would require a little *more* removed glass than a double-concave and would save time only if the flat side did not need a polish. While polishing only one surface would have worked for the low-power eyepieces, it might not serve for the astronomical oculars. Galileo would probably have had to return the piece to polishing, if not all the way to fine grinding, to remove microscopic surface roughness on the flat side.

The existence of the shopping list does not prove that Galileo used any artillery ball to make the astronomical eyepieces, but he had to use something. Grinding both sides on a 4 to 4.5-inch diameter known spherical tool is a straightforward extension of the spectacle-maker's art, and conjectures about other sources of smaller accurate spheres in those pre-machine-tool days is unneeded. Thus, the double-concave substitute lens seems to have been chosen deliberately with great care and historical perspective. In any case, the making of the weak objective lens was a great deal more difficult than the straightforward lens grinder's technology used to make the eyepiece, and is the greater mystery.

6.0 LAST THOUGHTS

Galileo took the low-powered technology of the Dutch inventors, adapting and extending it for the astronomical telescope. What he performed was the process that is known in modern times as *development*. Perhaps they didn't even have a word for such a concept in those days. I hope I have made the case in these pages that the rapid fabrication and quality of his instruments demanded that he also develop some other technologies. At the time, these methods were generally held as trade secrets but are today considered creditable or even patentable.

Let's look at what he achieved and what he may have achieved:

1) He drastically increased spyglass magnification by removing it from the realm of spectacle lenses (already discussed in Zik & Hon). This forced him to start making lenses himself.
2) In doing so, he must have developed lens-making techniques that would have been considered unusual at the time, making shallow optical surfaces that were accurate over much larger areas than others could achieve for well over a year.
3) He made instruments that worked with afocal exit beams smaller than the entrance pupil of the eye. Spyglasses (a least those that were comfortable to use) employ exit beams much bigger than the eye. He finally settled on an exit beam in the vicinity of 1.3 mm, neither too low for astronomical use nor too high for comfort.

[48] http://www.scitechantiques.com/Galileo-Telescope-Anomalies-optics-3-23-06/ (Fig. 1 in that page)

4) He may have used some primitive version of an optical test that allowed him to infer that the objective lens was not only acceptable, but gave positional error information leading to rework or the size and shape of an aperture stop.

5) Two ways increase the power to a useful range for astronomy: either the objective's focus is extended or the eyepiece's strength is increased. Galileo could not have known that resolution increases with aperture, but he must have realized that large openings and more light-gathering ability were associated with long telescopes. He chose this harder path.

It is unfortunate that Galileo reserved his published remarks to gross generalities such as magnification. Either he understood refractive optics well enough to design and construct his telescopes or he did not. Based on the good performance of his three surviving objectives, the quick schedule, and the letter to Clavius quoted earlier, I'm inclined to the simplest solution: he understood the crude optics of his time.

When he published *Sidereus Nuncius* in 1610, there were some targets that he intended to observe over the next year, including Mars, Saturn, and Venus. He may have wanted priority in any discoveries found on those planets, so he apparently hid what he knew about telescope making in *Nuncius*, replacing it with a lame tale on how he used pure logic and refraction theory to infer how the spyglass worked. He revealed nothing about his surely extensive methods of making the astronomical telescope. Early on he may have considered this his duty, since he didn't owe anything to his readers but to his patrons.[49] He only wanted to warn people so they would not try to confirm his results (and fail) with tiny spyglasses.

But why did he trot out the same tired story in the 1623 *Assayer*, when many others were producing astronomical telescopes without his help? Perhaps he was just keeping his options open in case the patronage of the Medici ceased. Then again, he may have avoided anything which made him resemble an artisan. He had bootstrapped himself into the position of mathematician and philosopher to the court of the Grand Duke of Tuscany. Common lens grinders did not move in the same circles as philosophers. The messy way he must have made and tested the objective lenses would have drawn attention to the fact that he had merely *crafted* the optics and that much of the process remained poorly understood.

Late in life, he might have relented somewhat on this reticence to share details of his lens-making,[50] but he did so privately or at the behest of his patron.

There is also the matter of what telescope discovered which celestial phenomena and whether the telescopes in the *Museo Galileo* are those telescopes. Clearly the 1710-mm focus isolated lens, having good aperture less than or equal to 38 mm, was large enough and of fine enough quality to have made many of the observations. It is mounted and honored as such, and there is no reason to doubt it. However, it probably did not exist at the beginning of the discoveries – he first mentioned it in a letter of 7 Jan 1610, and said that it was *almost* finished – so we must look at the other telescopes and ask whether they could have been involved in the discoveries.[51]

The 1330-mm focal length, 14-power telescope has a maximum aperture intermediate to the other two at 26 mm. Greco, *et al*., say the wavefront is at least 1/4 wavelength over this aperture. This makes it good enough for astronomy use. However, the 14-power eyepiece does not deliver high enough magnification for easy planetary work. The 16-mm aperture of the 980 mm focal-length objective was capable of the lunar observations, but my tests indicate that its dim image was not well adapted to Jupiter's moons. Such observations may have been possible, but they were not easy; it is intolerant of background light or lower

49 M. Biagioli, *Galileo's Instruments of Credit: Telescopes, Images, Secrecy*, p. 125, U. of Chicago Press, 2006.
50 Valleriani, *Galileo Engineer,* pp. 46-47 (2010).
51 S. Drake, *Galileo at Work, His Scientific Biography*, p. 147, Univ. of Chicago Press, 1978.

transparency. The 16-mm maximum aperture was also inadequate for the observations of the dimmest stars he reported.

And then there is the red leather and gold-leaf decoration on the 980 mm. I confess that this adornment bothers me most of all. What is its purpose? I may be arbitrarily imposing my values on a person who lived in an era and land completely foreign to me, but right there beside this embellished telescope sits the plain, undecorated 14x. It is sparsely but efficiently held together with wire windings (the duct tape of that age). If both telescopes were discovery instruments donated at the same time to the Medici collection or were later restored by a conservator who wanted to show them in the best light, why don't they both have the same level of decoration (or lack of it)?

The most likely origin of the adorned telescope was the group Galileo is known to have prepared for the Medici family to export to other princes or prelates in the post-*Nuncius* period. It would have been good enough for lunar views, which is all these lofty personages were likely to observe anyway. Its decoration was extra fancy, which redounded well to the ducal court. Either this telescope was never sent or one recipient didn't use it anymore. He (or his heirs) may have politely returned it to the collection. In a draft letter of March 19, 1610 Galileo wrote of a favorite instrument that he intended to give someday to the Grand Duke. This richly detailed instrument is definitely not the "plain and scruffy" telescope mentioned in that letter.[52]

At the time, Galileo apparently preferred a 20x for many of the raw observations, even if he didn't emphasize the fact in *Nuncius*.[53] And where is the 20 power used by Galileo if the one in the *Museo* is not it? I believe this telescope is in plain view, but it is not the obvious choice.

It would have been a simple matter to have made another tailpiece of the 14x, longer by about 30 mm and with a stronger negative lens.[54] This makes a 20x telescope with a 26-mm aperture and a full magnitude deeper light grasp than the 16 mm. At $\Delta n = 2$, it would have had less chromatic aberration than the full-bore 1710-mm. Its exit pupil would be moderate at 1.3 mm. Table A-3 has a tiny circle around the 16-arcminute field of view for a 7-mm eye pupil that would have allowed a good spread of Jupiter's moons to fit on most days.

I think that what is now called the "developmental" telescope was his early workhorse, and was finally donated by Galileo after he was done with it. These arguments could be also made about the broken 1710-mm lens with the missing ocular and tube, but the 1710 mm still has a more limited field-of-view. Figure 13 shows the maximum field of view with a 7-mm eye pupil at around 13 arcmin. More likely is 11 or 12 arcmin with the eye not pushed right against the glass (see Table A-2). If Galileo was using some slight illumination on his tube micrometer, his eye pupil – and hence the field – may have been even smaller.

Without further evidence, my hypothesis that the 14x is also the 20x instrument of December 1609 cannot be verified. If there were two tailpieces originally, it is no surprise that the loose one did not survive the 400 intervening years. However, I am thoroughly convinced after using the thumbnail-sized aperture on my simulated 20x/980mm that it is not the instrument in which Jupiter's satellites were easily noticed or further observed. The stellar and planetary observations had to be mostly made using a larger aperture than 16 mm.

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[52] D. Wootton, *Galileo, Watcher of the Skies*, p. 116, Yale Univ., 2010. Originally "...inornato et mal pulito..." from *Opere (works) di Galileo, Vol X*, p. 297 (Italian).

[53] S. Drake, *Galileo at Work*, read discussion on pp. 147-150, 1978.

[54] A double concave lens ground on a 5-inch ball from the 18-pound culverin cannon would have the correct focal length.

APPENDIX

A.1 CHROMATIC ABERRATION

Galileo did not have the benefit of 400 years of optical discovery, so he had to just try different lenses to find a good one, and then try different diaphragm openings until he found the brightest image that didn't show too much loss of contrast. I make the conjecture in this paper that he may have used an elementary test, but Figure 13 shows that he was not trying to eliminate residual chromatic aberration.

In the meantime, we have learned how to calculate the amount of tolerable aberration. First of all, note that the chief aberration left in even perfect spherical lenses is chromatic aberration; it is just a form of defocus for different colors. Blue is focused shorter than red, so it reaches focus closer to the objective lens. The typical way of stating chromatic aberration is to take the excess (over air) index of refraction in the yellow 588 nm and divide by the difference between the index of refraction at 486 and 656 nm (two easy-to-identify absorption lines in blue and red). The resultant number is the Abbe or V number for that type of glass.[55] For our soda-lime glass as read off an index chart, it is

$$V = (n_{yellow} - 1)/(n_{blue} - n_{red}) \;=\; (1.52305\text{-}1)/(1.5293\text{-}1.5204)\text{=}58.8. \qquad \text{(A-1)}$$

This is from reading off a graph. A more accurate calculation is 58.5. For the long, slow lenses being discussed here, the range of focus between blue and red is just

$$\Delta f \sim \frac{f_{obj}}{V} \qquad \text{(A-2)}$$

Having a 1-m focal length objective, we are focusing a distance of 17.1 mm between 486 nm and 656 nm, centered around 1 m. Taking the depth of focus expression again, and equating it to the expression above[56]

$$\Delta f = 8F^2 \Delta n\,\lambda = \frac{f_{obj}}{V} \quad \text{or} \quad 8\left(f_{obj}/D_{obj}\right)^2 \Delta n\,\lambda = \frac{f_{obj}}{V}$$
$$D_{obj} = \sqrt{8\, f_{obj}\, \Delta n\, \lambda\, V}\ .$$

This presents us with the puzzle of what to choose for Δn. It is not as if we can just set it equal to ±1/4 wave and go on calculating. The blue and red wavelengths of 486 and 656 nm were chosen because glass companies find them easier to measure, not because they correspond to wavefront measures. Let's choose Δn in analogy to another marginally color-corrected refractor. I once observed through a very fine

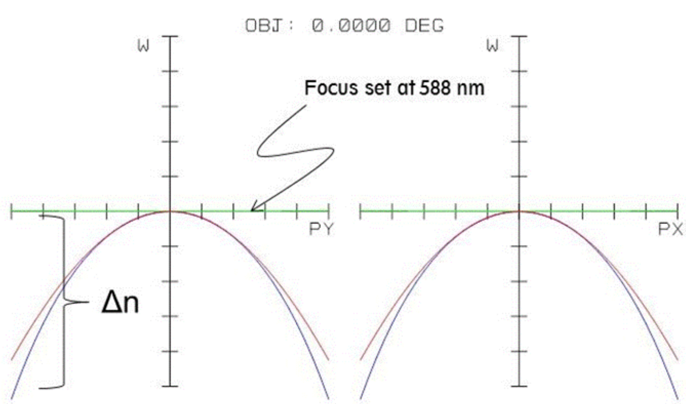

Figure A-1. Optical path differences at the same wavelengths used to calculate V in 6-inch f/15 Fraunhofer achromat. (scale = 1 wavelength)

55 R.E. Fischer and B. Tadic-Galeb, *Optical System Design*, pp. 99-101, SPIE Press, 2000.

56 H.R. Suiter, *Star Testing Astronomical Telescopes, 2nd Ed.*, App. E & p.84. Willmann-Bell, Inc. 2009.

6-inch *f*/15 Brashear-designed achromat attached as a guide telescope to the 16/25-inch Schottland Schmidt camera[57] when it was at Delaware, Ohio. The image was just beginning to show color error on bright objects. There was a non-objectionable, but noticeable, purple haze around Saturn, but deep-sky objects did not present easy to see color at all. If we look at the theoretical wavefront of a typical air-spaced achromat shown in Figure A-1, we see that in the red and blue, the aberration at the edges stretches to about Δn =1. Let's guess that value also applies to the Galilean telescope.

Thus, plugging all the numbers in for the equation above:

$$D_{obj} = \sqrt{8\, f_{obj}\, \Delta n\, \lambda\, V} = D_{obj} = \sqrt{8\, f\, [mm] 1\, (0.00056\, mm) 58.5} \;, \text{ or}$$

$$D_{obj} \leq \frac{\sqrt{f\,[mm]}}{2} \;. \tag{A-3}$$

This is plotted as the bottom line in Figure 13 in the main text. Anything in the hatched zone remains diffraction-limited, but choosing an aperture much below this line is just throwing light away. Strehl ratio from the ZEMAX using a spectrum of 17 wavelengths is 20x: 0.84, 14x: 0.63, and 30x: 0.46. Recall that the diffraction-limit cutoff for Strehl ratio is 0.8, so only the 20x is safely diffraction limited. It exactly meets the Strehl criterion at an aperture of 17.4 mm.

By the way, equivalents of the above formula for modern achromats and apochromats are:

$$D_{obj} \leq 3\sqrt{f\,[mm]} \quad \text{[achromat]} \tag{A-4}$$

$$D_{obj} \leq 4.5\sqrt{f\,[mm]} \quad \text{[apochromat]} \tag{A-5}$$

Thus, using the same Δn=1, a 1000-mm focal length achromatic lens can tolerate an aperture of about 95 mm (f/10.5) and an apochromat permits an aperture of about 140 mm (f/7).[58]

The diameter of the exit light bundle is 1.8 mm for the 14x, 0.8 mm for the 20x, and 1.26 mm for the 30x. These are all small; the 20x exit bundle is on the near edge of what is useful power. A 0.8 mm exit bundle is 1441 wavelengths across, giving a maximum spatial frequency of 1441 cycles/rad or 0.42 cyc/arcmin. This yields a resolution of 2.4 arcmin, larger than the native 20/20 resolution of the eye (1.25 arcmin, or thereabouts). If Galileo had excellent vision, he was capable of seeing a small amount of this diffraction blur. By contrast, a typical 20x80mm astronomical binocular has an exit pupil of 4 mm, and its optics are typically limited by the vision of the eyes.

If the wavelengths 486 and 656 nm are not better places to indicate the very edges of the diffraction limit, what wavelengths are? Lumping the photopic sensitivity curve of the eye into three equally-weighted bins located at 510, 560, and 610 nm, a very good approximation is achieved.[59] Figure A-2 demonstrates this approximation in the center of a set of three ZEMAX geometric spot diagrams. (The half-spots are right at the edge of the vignetting pattern of Figure 4 where the throughput is 50%. The rest of the fields have been trimmed away.)

The figure labeled "20x" shows the spots for 0.51 and 0.61 micrometers (that is, 510 and 610 nm) just fill the Airy disk limits. The Airy disk limits for the 14 and 30 power telescopes are calculated for the apparent aperture chosen by Galileo instead of the ones on the lower line of Figure 13. If these lenses were achromats or better, the spot patterns would just barely fill at 0.51 and 0.61 microns. The spots would not appear more compact, but the Airy disks would swell to encompass the spot. The 14x would work a little better because its chromatic aberration is not magnified so greatly, but the 30x would

57 H.C. King, *The History of the Telescope*, p. 370, 1955 (Dover, 1979).
58 Adapted from H. Suiter, *Star Testing Astronomical Telescopes, 2nd Ed.*, from expressions 12.1 and 12.2, Willmann-Bell, 2009
59 Bruce H. Walker, *Optical Design for Visual Systems*, p. 5, TT45 SPIE Press, 2000.

definitely look a little fuzzy. Note, however, that the 14x works well because its magnification is insufficient to expand the blur spots (in angle at the eye) beyond the $\Delta n = 1$ case of the 20x.

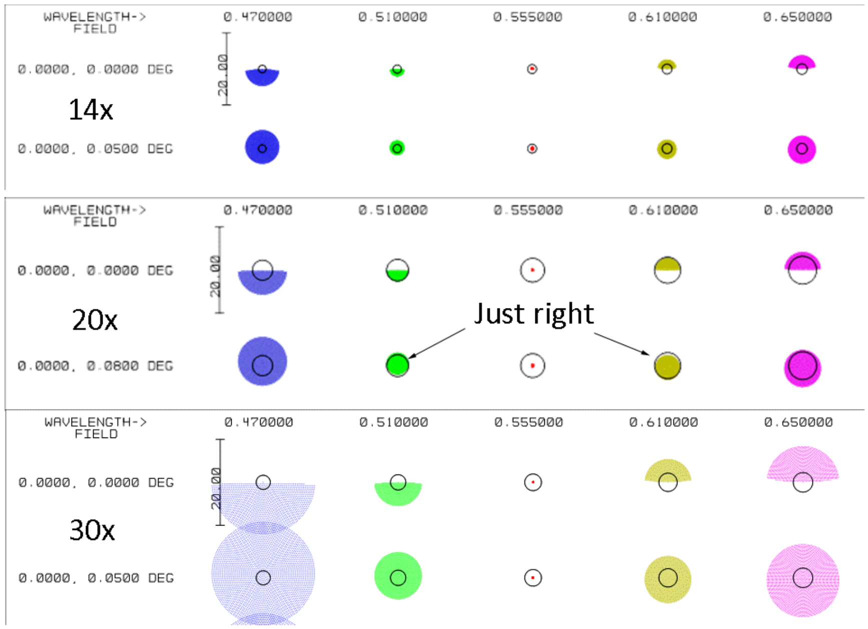

Figure A-2. Spot sizes in the three Galileo refractors using the aperture stops that have survived.
Only the 20x works to modern achromatic standards. Scale is afocal arcminutes at eye.
Half-illuminated disks are at the edge of the pupil as in Figure 5.

A.2 LIKELY RESOLUTION OF GALILEAN INSTRUMENTS

Continuing the assumption that the Galilean instruments were defective primarily because of chromatic aberration, let's ask what the resolution was. For the 20x, I take an oversampled distribution of wavelengths and calculate the grayscale diffraction pattern in ZEMAX. It is shown in Figure A-3. Please keep in mind that the central spot should be yellow green, surrounded by a purple glow. This is not the centered pattern but the typical pattern that would be seen at 1.6 mm off center in the eyepiece, but still close to the middle.

The resolution may be inferred by looking at the photopic distribution's modulation transfer function (MTF) in Figure A-4. By following the drop forward to the axis crossing of 0.24 cycles/arcminute, we read off that the resolution is better than 1/0.24 ~ 4 arcmins. Keep in mind that this is an afocal beam and it includes the effect of the eyepiece. This result is corroborated by the encircled-energy function. Setting an arbitrary threshold such as 0.8 (the same as the Strehl ratio), the pattern can be seen to cross at 3.6 arcminutes or so. To calculate the resolution in the sky, take 4 arcminutes (or 240 arcseconds) and divide it by 20 power to yield 12 arcseconds.

Let's look at the 30x in another case with the eye a little farther off axis. The diffraction pattern is shown in Figure A-5. Recall that this aperture allows more chromatic aberration, with $\Delta n \sim 3$ instead of 1 as it was for the 20x. The centered Strehl ratio will creep up to 0.46, but it is not as high as the $\Delta n = 1$ twenty power instrument. At 0.15 degrees off axis, it falls to 0.38, but the system is just too slow to exhibit much change with field radius. In the case of the 30-power's aperture, sharpness has been traded for image brightness. Figure A-6 shows the fundamental resolution of the whole afocal system is about 5 arcminutes, which translates to about 300 arcsec/30 = 10 arcsec in the sky. This indicates once again that aperture can often be used to defeat imperfections. In this case, the more severe aberration of the 37 or 38-mm clear aperture of the 30x resolves a little better than the 16 mm aperture of the 20x.

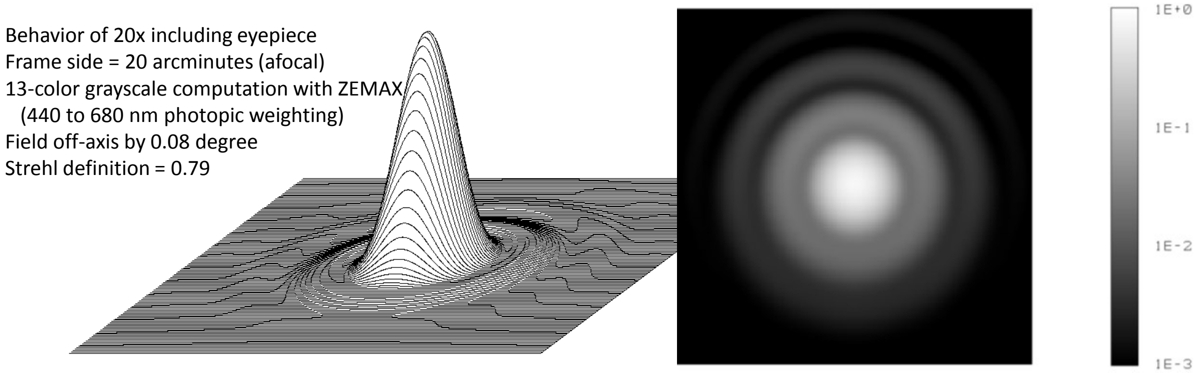

Figure A-3. The diffraction pattern of the 20x at the eye. At right is the log diffraction pattern.

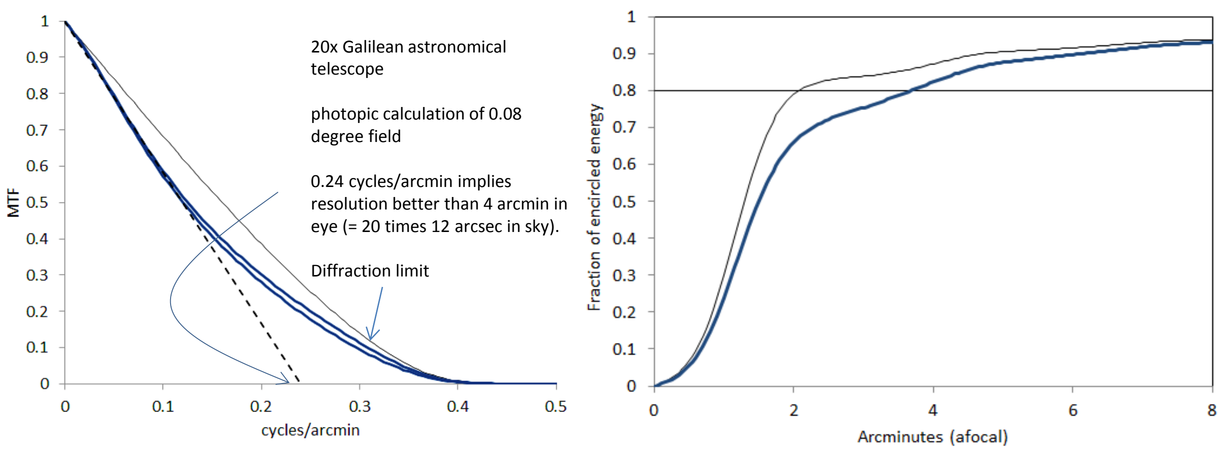

Figure A-4. MTF and EE fraction for 20x Galilean. Calculated for afocal beam.

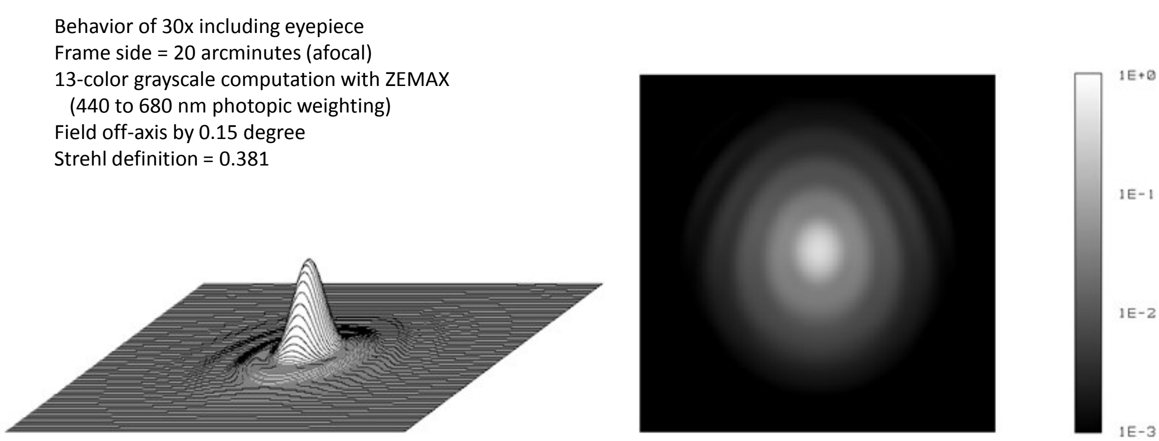

Figure A-5. Off-axis diffraction spot of the 30x is stretched slightly by astigmatism, but is otherwise quite similar to the on-axis case.

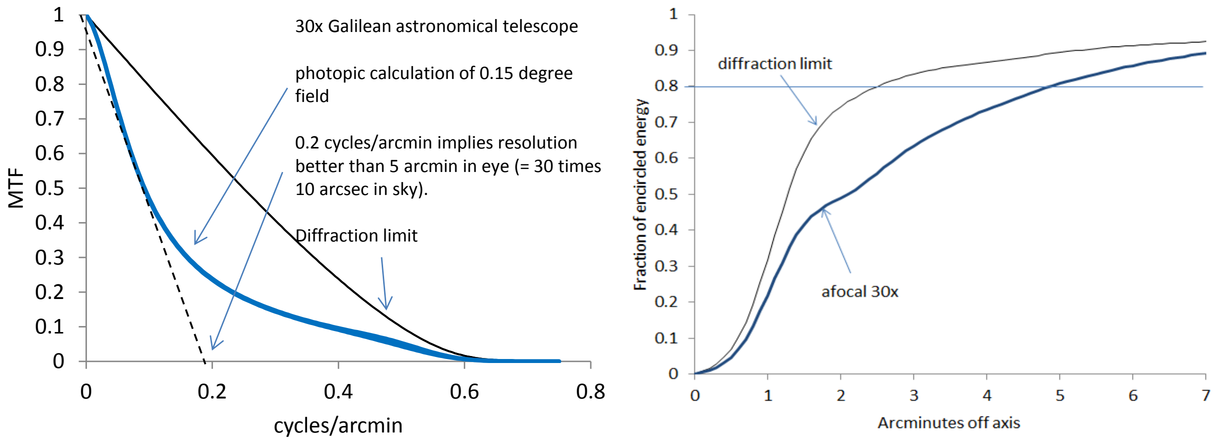

Figure A-6. Inferring the resolution of the 30x Galilean discovery instrument.

How does this compare with resolutions inferred from Galileo's observations? One researcher suggests from internal evidence that Galileo's angular resolution may have been as bad as 20 arcseconds.[60] Any scattering centers or lemon-peel finish induced by the lap material scatters light into a broad glow and only affects point-to-point resolution in an indirect manner. Even fairly bad micro-roughness or diffractive scattering will have the effect of decreasing the entire MTF curve uniformly, which changes the zero intercept of the curve little. Another way of calculating it is to shrink the encircled energy by a bad fraction, say the 6-percent tolerance for bad veiling glare.[61] The resolution of the 30x afocal system, defined by the encircled-energy line at 0.8, increases to about 6 arcminutes. Dividing by 30 still is only 12 arcseconds. To expand the resolution to 20 arcseconds is the same as assuming that there was another profound aberration source much larger than chromatism. Even at Δn = 3.2, chromatism is large but not enough to account for the added aberration. Geometric aberrations of this magnitude did not appear in the Greco data. The extra aberration does not seem to be explainable by assuming only instrument error.

A.3 FIELD OF VIEW AND MAGNIFICATION OF GALILEAN TELESCOPES

Let's list some other interesting quantities of Galilean telescopes (found in Table A-1 through A-3). In the upper left corner there is the field of view. We have briefly mentioned how this is calculated, but let's discuss this in more detail. It is calculated not by the usual method of Keplerian telescopes:

$$FOV_{Kep} = D_{field\ stop} / f_{obj} \ , \tag{A-6}$$

but as the diameter of the eye's pupil P as viewed from the exit pupil's distance

$$apparent\ FOV_{Gal} = P / d_{exit\ pupil}. \tag{A-7}$$

The distance of the exit pupil can be calculated by viewing it as an image of the objective as seen in the eyepiece, and figuring it out with the thin-lens equation.

$$\frac{1}{i} + \frac{1}{d_{obj}} = \frac{1}{f_{eye}} \tag{A-8}$$

Solving for image distance from lens i yields expression (for high powers)

$$i \cong f_{eye}(1 - {}^{1}\!/_{m}) \ ,$$

60 E. Bernieri, "Learning from Galileo's errors," *J. Br. Astron. Assoc.*, pg. 171, 122, 3, 2012.

61 H. Kondo, *et al.*, "Simplified method for measuring and assessing the veiling glare of 35 mm camera lenses," *Image Assessment: Infrared and Visible, Proc. SPIE, vol. 467*, pp. 62-65 (1983)

with m being the magnification.[62] The distance to the pupil of the eye is a few mm beyond this image distance, or the distance of the eye-glass separation will be called the "eye relief," even though it does not meet the formal definition:

$$d_{exit\ pupil} = f_{eye}\left(1 - {}^{1}\!/_{m}\right) + e.r. \tag{A-9}$$

Divided into the eye pupil's size, this is the *apparent* field-of-view. To derive true field-of-view in the sky, divide by the power. As an example, let's do the calculation for a generic 1000 mm 20-power telescope into a 7-mm eye pupil with 10 mm of glass-to-eye separation.

$$P_{eye}/d_{exit\ pupil} = 7\ /\{50(1–1/20)+10\} = apparent\ FOV = 7/57.5 = 0.122 = 6.9\text{ deg}$$
$$sky\ FOV = 6.9/20 = 0.34\text{ deg} = 20.8\text{ arcmin}$$

Let's do an approximation of this expression. First of all, assume the eye relief is small compared to the focal length of the eyepiece. Then assume the magnification is much greater than one:

$$sky\ FOV = P_{eye}/\left[f_{eye}\left(1 - {}^{1}\!/_{m}\right) + e.r.\right]/m \cong mP_{eye}/[f_{eye}(m-1)]/m \cong [P_{eye}/f_{eye}]/m \tag{A-10}$$

Rewriting the common expression for magnification, $m\, f_{eye} = f_{obj}$,

$$sky\ FOV \cong P_{eye}/f_{obj}\ . \tag{A-11}$$

To first order the *sky FOV* of a high power astronomy-type Galilean does not depend on anything other than the size of the entrance pupil of the eye and the focal length of the instrument. Let's try it: 7 mm/ 1000 mm = 24.1 arcmin seems to underestimate the *e.r.* = 0 solution of 25.33 arcmin slightly, but it is a reasonable two number approximation.

We might try modifications to cure the tiny fields. For example, a too quick look at equation A-9 might convince us that decreasing the eyepiece focal length might bring the exit pupil closer. However, this also increases the counterbalancing magnification. Even though the apparent field does go up, the real field as measured in the sky goes down somewhat (look at the next section's tables). The only thing to do to increase the true field of view is to go to spyglass-type instruments and the Figure 5 right-side way of figuring FOV. Once the entrance pupil is pushed smaller than the front face of the objective lens, wider field is achieved. Galileo, however, was driven toward powers sufficient to completely use the clear aperture.

This condition limits the length of the Galilean astronomical telescope to about 2.5 m having a FOV somewhat less than 9.6 arcmin even with a 7 mm eye pupil pushed right against the glass. The field of view vanishes with long instruments and there isn't anything to do about it.

A.4 FOV CALCULATIONS FOR VARIOUS APERTURES AND FOCAL LENGTHS

Let's look at Table A-1 and point out interesting features of it. First of all, note that the FOV values apply only if the eye is placed 10 mm behind the glass and has a 7 mm pupil. Most of the blocks in this table are for variation of two parameters, the focal length of the objective along the green bar at the top and the focal length of the eyepiece in green bars down the side. The objective has focal lengths from 347 mm to 2722 mm in √2 jumps (selected so that the highlighted value has the correct entry). The eyepiece varies from 34 to 78 mm in 1/8th jumps, Simplest is the magnification table, about halfway down the left column. The 980 mm objective divided by the 48 mm eyepiece gives 20.4 power, as expected. The 20x values are denoted by a purple background.

[62] E. Roger Cowley, crab.rutgers.edu/~cowley/Galileo/galileo.htm 2009.

The beam diameters are calculated for transmission through the ideal apertures in the $\Delta n = 1$ blue line of Figure 13 (listed in the pink bar of Table A-1). Recall that the lowest power squeezing all of the light in the eye occurs for a 7-mm beam and the traditional highest power is somewhere around a 0.5-mm beam. It is the highest practical power because the fuzz of diffraction becomes large enough to readily see. The lower left-hand values are nowhere near 7 mm, but the upper right does get smaller than 0.5 mm. Exit beams sizes are useless.

At the lower right the apertures are listed again, ranging from an optimum value of 9.53 mm for the 347 mm focal length lens to nearly 27 mm for the 2772 mm focal length simple lens. The corresponding resolutions appear below, and the comparison with the typical resolution of a 20/20 eye (about 1.25 arcminutes) in the last row to the right. Please draw your attention to the 245 mm focal length, 8 mm aperture. These entries state that the 9.5 mm aperture ideally resolves points separated by only 14.8 arcseconds, which is 5 times better than human vision! What is happening when the 9.5-mm telescope is resolving over 5 times as well as a 7-mm eye?

The answer is that the human eye is far from diffraction-limited, but well-made telescopes of this aperture ratio are close to the ideal. We still have to get the light through the same substandard eye, but we do so in a much smaller beam bundle. Power helps avoid the innate aberration of the eye by using a smaller area of the eye's aperture. Of course, ideal resolutions such as these aren't seen in Galileo's real telescopes (see the section on resolution).

The next interesting feature of this table can be found in the block to the upper right. It states that for the example 980 mm, 20.4 power, instrument, used with 10 mm of eyelash room and with an iris opening of 7 mm, the apparent field of view is 7.2 degrees. By "apparent" it is meant the view in the eyepiece occupies this angle. If we multiply by 60, we calculate the table halfway down on the right, the apparent field in arcminutes. Now, take this number and divide it by the magnification on the left side, and we calculate the real field of view in arcminutes as seen in the sky. This table appears in the upper left of Table A-1. For the 980 mm 20.4x, we get 21.1, or about two-thirds the size of the Moon.

Looking at the Moon, however, would cause the eye to contract. For example, a 5 mm opening in the eye yields (5/7)*(21.1) = 15.1 arcmin. In the daytime, the iris shrinks to about 3 mm, yielding (3/7)*(21.1) = 9 arcmin. It makes finding and tracking objects difficult, and is barely usable even at night. It seems that the field might go away altogether continuing with this simple ratio, but minimum field even through a pinhole is:

$$skyFOV_{min} = (D_{exit\ pupil}/d_{exit\ pupil})/m \ . \tag{A-12}$$

where D is for the diameter of the exit pupil. For the 20x, this is 2.5 arcminutes, which is related to the falloff at the edges of Figure 6 of the main document.

Table A-2 shows the equivalent calculation using (yellow) input values around the 30x discovery instrument and Table A-3 is for the 14x development telescope.

Table A-1. Galilean telescope information as a function of objective and eyepiece focal length (purple is about Galileo's 20x) Eye relief is 10 mm and eye's pupil is 7mm. Δn=1

eye relief =	10
eye pupil =	7
n =	1

calculates FOV (arcminutes)

		f_{obj} [mm]						
		347	490	693	980	1386	1960	2772
	34	57.4	39.7	27.6	19.3	13.5	9.5	6.7
	38	59.7	41.1	28.5	19.9	13.9	9.8	6.9
	43	62.1	42.5	29.4	20.5	14.3	10.1	7.1
f_{eye} [mm]	48	64.6	44.0	30.3	21.1	14.7	10.3	7.3
	54	67.2	45.5	31.2	21.6	15.1	10.6	7.4
	61	70.0	47.0	32.1	22.2	15.4	10.8	7.6
	69	73.0	48.6	33.1	22.8	15.8	11.0	7.7
	77	76.3	50.4	34.0	23.3	16.1	11.2	7.9

calculates apparent field (degrees)

		f_{obj} [mm]						
		347	490	693	980	1386	1960	2772
	33.8	9.8	9.6	9.4	9.3	9.3	9.2	9.2
	38	9.1	8.8	8.7	8.6	8.5	8.4	8.4
	43	8.4	8.1	7.9	7.8	7.7	7.7	7.6
f_{eye} [mm]	48	7.7	7.5	7.3	7.2	7.1	7.0	7.0
	54	7.2	6.9	6.7	6.5	6.4	6.4	6.3
	61	6.6	6.3	6.1	6.0	5.9	5.8	5.7
	69	6.1	5.8	5.6	5.4	5.3	5.3	5.2
	77	5.7	5.3	5.1	4.9	4.8	4.8	4.7

calculates magnification

		f_{obj} [mm]						
		347	490	693	980	1386	1960	2772
	33.8	10.3	14.5	20.5	29.0	41.0	58.0	82.0
	38	9.1	12.9	18.2	25.8	36.4	51.5	72.9
	43	8.1	11.5	16.2	22.9	32.4	45.8	64.8
f_{eye} [mm]	48	7.2	10.2	14.4	20.4	28.8	40.7	57.6
	54	6.4	9.1	12.8	18.1	25.6	36.2	51.2
	61	5.7	8.0	11.4	16.1	22.8	32.2	45.5
	69	5.1	7.2	10.1	14.3	20.2	28.6	40.5
	77	4.5	6.4	9.0	12.7	18.0	25.4	36.0

calculates apparent field (arcmin)

		f_{obj} [mm]						
		347	490	693	980	1386	1960	2772
	33.8	588	575	566	559	555	552	550
	38	544	530	520	513	508	505	503
	43	503	487	477	470	465	461	459
f_{eye} [mm]	48	465	448	437	429	424	420	418
	54	430	412	400	392	386	383	380
	61	398	378	366	357	351	347	345
	69	369	348	334	325	319	315	312
	77	343	320	306	296	290	286	283

apertures [mm]		9.53	11.33	13.48	16.03	19.06	22.67	26.95

calculates beam diameter (mm)

		f_{obj} [mm]						
		347	490	693	980	1386	1960	2772
	33.8	0.93	0.78	0.66	0.55	0.46	0.39	0.33
	38	1.05	0.88	0.74	0.62	0.52	0.44	0.37
	43	1.18	0.99	0.83	0.70	0.59	0.49	0.42
f_{eye} [mm]	48	1.32	1.11	0.94	0.79	0.66	0.56	0.47
	54	1.49	1.25	1.05	0.89	0.74	0.63	0.53
	61	1.68	1.41	1.18	1.00	0.84	0.70	0.59
	69	1.88	1.58	1.33	1.12	0.94	0.79	0.67

		f_{obj} [mm]						
apertures [mm]		9.53	11.33	13.48	16.03	19.06	22.67	26.95

calculates resolution (arcsec)

		347	490	693	980	1386	1960	2772
	any e.p.	14.8	12.4	10.5	8.8	7.4	6.2	5.2
1.25	compare	5.1	6.0	7.2	8.5	10.1	12.1	14.3
arcmin								

Table A-2. Galilean telescope information as a function of objective and eyepiece focal length (purple is about Galileo's 30x) Eye relief is 10 mm and eye's pupil is 7mm. Δn=3.2

eye relief = 10

eye pupil = 7

n = 3.2

calculates FOV (arcminutes)

f_{eye} [mm]	f_{obj} [mm] 605	855	1209	1710	2418	3420	4836
37.95	32.9	22.9	16.0	11.3	7.9	5.6	3.9
43	34.0	23.6	16.5	11.6	8.1	5.7	4.0
48	35.1	24.3	16.9	11.9	8.3	5.9	4.1
54	36.2	25.0	17.4	12.2	8.5	6.0	4.2
61	37.3	25.6	17.8	12.4	8.7	6.1	4.3
68	38.4	26.3	18.2	12.7	8.9	6.2	4.4
77	39.6	27.0	18.6	12.9	9.0	6.3	4.5
87	40.8	27.7	19.0	13.2	9.2	6.4	4.5

calculates apparent field (degrees)

f_{eye} [mm]	f_{obj} [mm] 605	855	1209	1710	2418	3420	4836
37.95	8.7	8.6	8.5	8.5	8.4	8.4	8.4
43	8.0	7.9	7.8	7.7	7.7	7.6	7.6
48	7.4	7.2	7.1	7.0	7.0	7.0	6.9
54	6.7	6.6	6.5	6.4	6.4	6.3	6.3
61	6.2	6.0	5.9	5.8	5.8	5.7	5.7
68	5.7	5.5	5.4	5.3	5.2	5.2	5.2
77	5.2	5.0	4.9	4.8	4.7	4.7	4.7
87	4.8	4.6	4.4	4.3	4.3	4.2	4.2

calculates magnification

f_{eye} [mm]	f_{obj} [mm] 605	855	1209	1710	2418	3420	4836
37.95	15.9	22.5	31.9	45.1	63.7	90.1	127.4
43	14.2	20.0	28.3	40.0	56.6	80.1	113.3
48	12.6	17.8	25.2	35.6	50.3	71.2	100.7
54	11.2	15.8	22.4	31.6	44.7	63.3	89.5
61	9.9	14.1	19.9	28.1	39.8	56.3	79.6
68	8.8	12.5	17.7	25.0	35.4	50.0	70.7
77	7.9	11.1	15.7	22.2	31.4	44.4	62.9
87	7.0	9.9	14.0	19.8	27.9	39.5	55.9

calculates apparent field (arcmin)

f_{eye} [mm]	f_{obj} [mm] 605	855	1209	1710	2418	3420	4836
37.95	524	516	511	507	505	503	501
43	481	473	467	463	461	459	457
48	441	433	427	422	420	417	416
54	405	395	389	384	381	379	378
61	371	361	354	349	346	344	343
68	340	329	322	317	314	312	310
77	311	300	293	288	284	282	280
87	285	274	266	261	257	255	253

calculates beam diameter (mm)

apertures [mm]	22.52	26.78	31.84	37.87	45.03	53.55	63.68
f_{eye} [mm]	f_{obj} [mm] 605	855	1209	1710	2418	3420	4836
37.95	1.41	1.19	1.00	0.84	0.71	0.59	0.50
43	1.59	1.34	1.12	0.95	0.80	0.67	0.56
48	1.79	1.50	1.27	1.06	0.89	0.75	0.63
54	2.01	1.69	1.42	1.20	1.01	0.85	0.71
61	2.26	1.90	1.60	1.35	1.13	0.95	0.80
68	2.55	2.14	1.80	1.51	1.27	1.07	0.90
77	2.87	2.41	2.03	1.70	1.43	1.20	1.01
87	3.22	2.71	2.28	1.92	1.61	1.36	1.14

calculates resolution (arcsec)

apertures [mm]		22.52	26.78	31.84	37.87	45.03	53.55	63.68
		f_{obj} [mm] 605	855	1209	1710	2418	3420	4836
	any e.p.	6.3	5.3	4.4	3.7	3.1	2.6	2.2
1.25 arcmin	compare	12.0	14.3	16.9	20.2	24.0	28.5	33.9

Table A-3. Galilean telescope information as a function of objective and eyepiece focal length (purple is about Galileo's 14x) Eye relief is 10 mm and eye's pupil is 7mm. Δn=1.94

eye relief =	10
eye pupil =	7
n =	1.94

calculates FOV (arcminutes)

		f_{obj} [mm]						
		472	668	944	1335	1888	2670	3776
	64	49.8	33.9	23.3	16.2	11.3	7.9	5.6
	72	51.5	34.9	23.9	16.6	11.5	8.1	5.7
	81	53.4	35.9	24.5	16.9	11.8	8.2	5.8
f_{eye} [mm]	91	55.5	37.0	25.1	17.3	12.0	8.4	5.9
	103	57.8	38.1	25.7	17.6	12.2	8.5	5.9
	115	60.4	39.4	26.4	18.0	12.4	8.6	6.0
	130	63.5	40.8	27.1	18.4	12.6	8.8	6.1
	146	67.1	42.4	27.9	18.8	12.8	8.9	6.2

calculates apparent field (degrees)

		f_{obj} [mm]						
		472	668	944	1335	1888	2670	3776
	64	6.1	5.9	5.7	5.6	5.6	5.5	5.5
	72	5.6	5.4	5.2	5.1	5.0	5.0	5.0
	81	5.2	4.9	4.8	4.6	4.6	4.5	4.5
f_{eye} [mm]	91	4.8	4.5	4.3	4.2	4.1	4.1	4.0
	103	4.4	4.1	3.9	3.8	3.7	3.7	3.7
	115	4.1	3.8	3.6	3.5	3.4	3.3	3.3
	130	3.8	3.5	3.3	3.2	3.1	3.0	3.0
	146	3.6	3.2	3.0	2.9	2.8	2.7	2.7

calculates magnification

		f_{obj} [mm]						
		472	668	944	1335	1888	2670	3776
	64	7.4	10.4	14.8	20.9	29.5	41.7	59.0
	72	6.6	9.3	13.1	18.5	26.2	37.1	52.4
	81	5.8	8.2	11.7	16.5	23.3	33.0	46.6
f_{eye} [mm]	91	5.2	7.3	10.4	14.7	20.7	29.3	41.4
	103	4.6	6.5	9.2	13.0	18.4	26.0	36.8
	115	4.1	5.8	8.2	11.6	16.4	23.2	32.7
	130	3.6	5.1	7.3	10.3	14.6	20.6	29.1
	146	3.2	4.6	6.5	9.1	12.9	18.3	25.9

calculates apparent field (arcmin)

		f_{obj} [mm]						
		472	668	944	1335	1888	2670	3776
	64	367	353	344	338	334	331	329
	72	338	323	314	307	303	300	298
	81	311	296	286	279	274	271	269
f_{eye} [mm]	91	287	271	260	253	248	245	243
	103	266	248	237	230	225	221	219
	115	247	228	216	208	203	200	197
	130	231	210	197	189	184	180	178
	146	217	194	180	172	166	162	160

apertures [mm]		15.49	18.42	21.91	26.05	30.98	36.84	43.82

calculates beam diameter (mm)

		f_{obj} [mm]						
		472	668	944	1335	1888	2670	3776
	64	2.10	1.77	1.49	1.25	1.05	0.88	0.74
	72	2.36	1.99	1.67	1.41	1.18	0.99	0.84
	81	2.66	2.24	1.88	1.58	1.33	1.12	0.94
f_{eye} [mm]	91	2.99	2.51	2.11	1.78	1.50	1.26	1.06
	103	3.36	2.83	2.38	2.00	1.68	1.41	1.19
	115	3.79	3.18	2.68	2.25	1.89	1.59	1.34
	130	4.26	3.58	3.01	2.53	2.13	1.79	1.51
	146	4.79	4.03	3.39	2.85	2.40	2.01	1.69

f_{obj} [mm]

apertures [mm]		15.49	18.42	21.91	26.05	30.98	36.84	43.82

calculates resolution (arcsec)

		472	668	944	1335	1888	2670	3776
	any e.p.	9.1	7.6	6.4	5.4	4.5	3.8	3.2
1.25	compare	8.2	9.8	11.7	13.9	16.5	19.6	23.3
arcmin								

A.5 INTERPRETING RONCHI SCREEN RESULTS

Taking a geometric expression for the Ronchi result used to test telescope mirrors and adapting it for the somewhat more tolerant 3rd order spherical aberration: [63,64]

$$\delta = \left(\frac{2}{1.22}\right)\frac{6 \cdot 2.44\lambda}{n}\frac{f}{2r} = 12\frac{\lambda f N}{rn}. \qquad \text{(A-11)}$$

In this expression, δ is the distortion, λ the wavelength, N the number of line-pairs (or periods) per length unit in the grating, n the number of periods showing on the area of regard, and r the radius of area of regard. For 3 line pairs on the disk at 0.00056 mm wavelength, the expression becomes

$$\delta = \frac{1}{445}\frac{fN[\text{in } lp/mm]}{r}. \qquad \text{(A-12)}$$

Thus, examining the central 15 mm radius of a lens of 950 mm focal length using a grating of 1.5 lp/mm, permits a distortion of the lines of 1.5 (950/445)/15 = 0.213 ≅ 21%. This distortion is measured along a bar about halfway out the considered radius of the disk. If the lines bow out or are pinched in this amount, the wavefront only barely meets the diffraction limit in the considered area.

A.6 BUILDING A "DUTCH" SPYGLASS

I decided to build a simulated spyglass to test some of the field-of-view statements in the main document. Using an electrical tubing fitting called an extender and the same ring mounting as has been described already, I obtained a compact and satisfactory monocular using a surplus 300-mm object glass. For the eyepiece, the –105 mm lens was transferred from the main telescope to the spyglass. (Using the shorter eyepiece offered slightly greater in-travel, which was close to being absent in the extender.) The complete instrument is only 9 inches long, including eyecups and end shade, and has a diameter of about 2 inches. The native powers of its lenses are 3.3 and –9.5 diopters. It would have been somewhat shorter than instruments made with spectacle lenses, but not greatly so. It is shown in Figure A-7. The front of the instrument is not stopped down yet in the photograph at the right, but the rear is tightly constrained around the 7 mm at the center of the eyepiece. If the front is stopped down to 13 mm, it displays a field of less than 1.5 degrees and shows no obvious color. I decided to leave it open, allowing more color at the same time as an increase in field angle. It is easy to point, a pleasure to use, and scarcely needs adjustment of focus. It is no wonder that people in 1609 were so eager to obtain one.

Figure A-7. The Dutch spyglass. The white-painted fence spans about a 3 degree real and a 9 degree apparent angle. Some chromatic aberration is visible, but I tend to seek the sweet spot near the center and aim the telescope accordingly.

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63 H. Suiter, *Star Testing Astronomical Telescopes, 2nd Ed.*, equation A-3, Willmann-Bell, 2009

64 P. Yoder, Jr., F. Patrick, & A. Gee, "Permitted Tolerance on Percent Correction of Paraboloidal Mirrors," *JOSA, vol. 43, no. 8*, pp. 702-703, Aug. 1953.